

1. IN ROD C ION

(1974), (1999), (1995), (1986), (1987), (1993), (1998), (1998), (2000), (2004),

(2002, 2005), (2002), (1998), (2001)

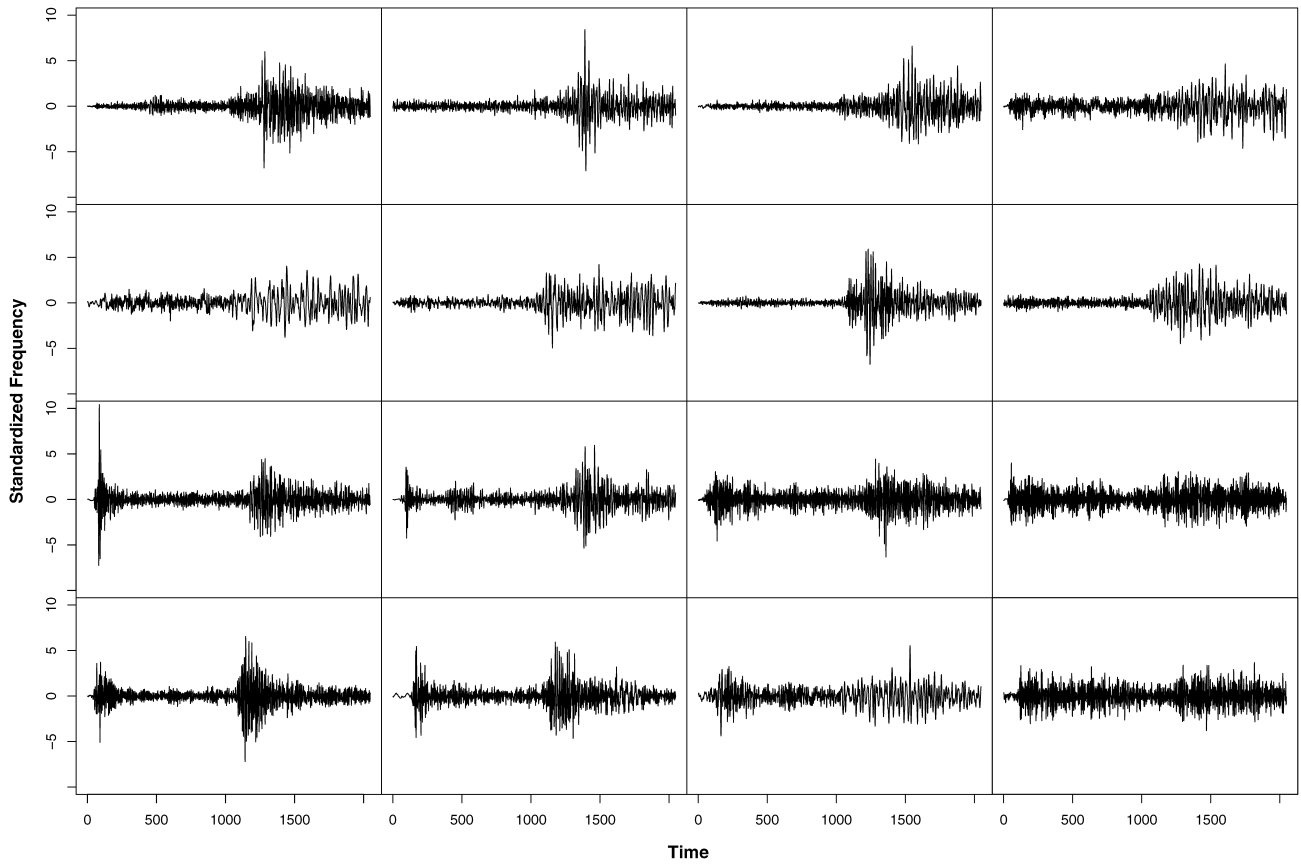
$$Y_j = g(t_j) + e_j(t_j), \quad j = 1, \dots, J,$$

$$v(t_j) = \dots (e_j(t_j))$$

$$v(x) = E(Y^2 | X=x) - E(Y | X=x)^2.$$

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 -03-54448, -05-05537.

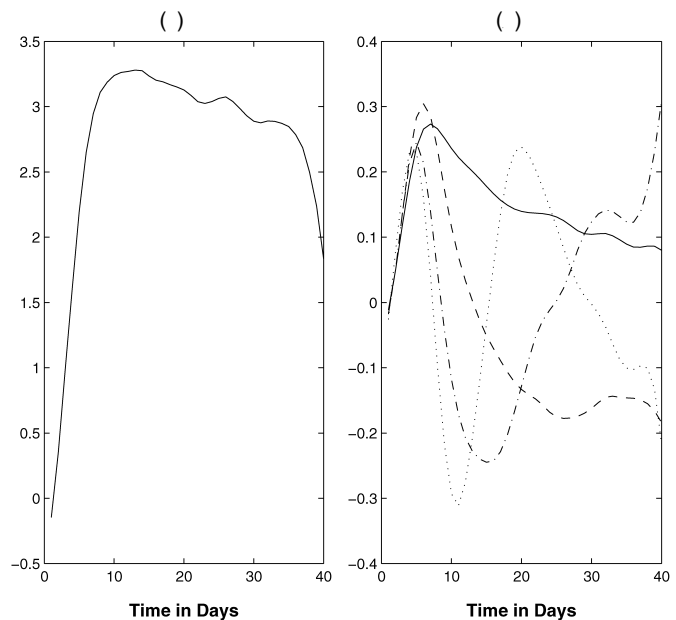
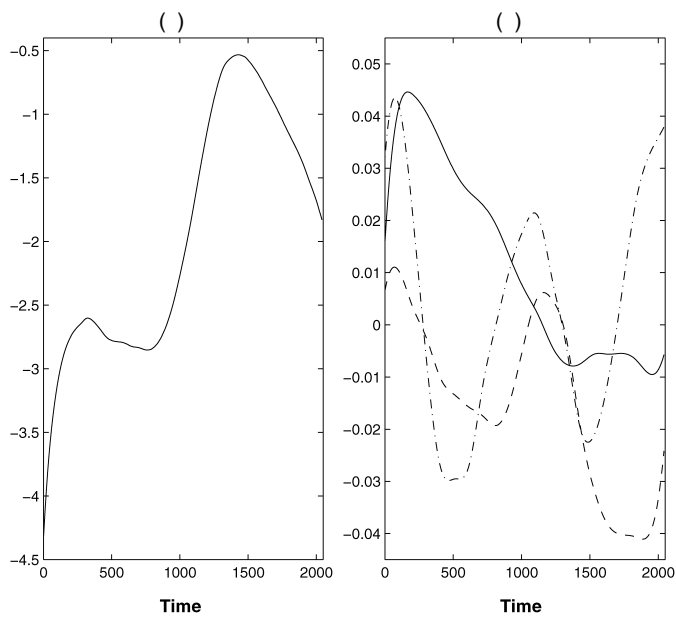
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F_{ij} 1. D E_{ij} E (n) , E (n) $.025$ n

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R_{ij} i
 S
 $= a_1, a_2$
 m
 (2.5)
 R_{ij}

F_i $2. C_n$ F P
 E E D $()$ $()$ E_i-L D
 $()$ $()$ $()$ n m n $()$
 $()$ $()$ n m $()$,
 $(- - -)$, $(- - -)$ $()$
 62.8% , 23.6% , 7.8%
 $()$ n (m) $()$, $(- - -)$,
 $(- - -)$, 48.3% , 21.0% , 11.6% , 6.7%
 m $.025$ $()$ $()$, $()$ $()$

a_1, a_2 S
 t_{ij}

$$X_{ij} = S_i(t_{ij}) + R_{ij}, \quad i = 1, \dots, n, j = 1, \dots, m. \quad (1)$$

R_{ij} R_{ij} R_{ik}
 $i \neq i'$

$$ER_{ij} = 0, \quad (R_{ij}) = \frac{2}{R_{ij}} < \infty.$$

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$$(Z_{ij}, Z_{ik}) = (V_i(t_{ij}), V_i(t_{ik})) = G_V(t_{ij}, t_{ik}), \quad j \neq k. \quad (9)$$

where G_V is the covariance function of V_i (1998) (2005). μ_V is the mean of V_i (1991).

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} k k(t) \quad (10)$$

$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} k k(t).$$

$$W_{ij} = \begin{cases} V_i(t_{ij}) + W_{ij} & \text{if } i = 1, \dots, n, j = 1, \dots, m, \\ 0 & \text{otherwise} \end{cases} \quad E(W_{ij}) = \frac{2}{W},$$

$$P(W_{ij} > 0) = P(W_{ij} < 0) = \frac{1}{2}. \quad X_{ij} = S_i(t_{ij}) + V_i(t_{ij}) + W_{ij} \quad (11)$$

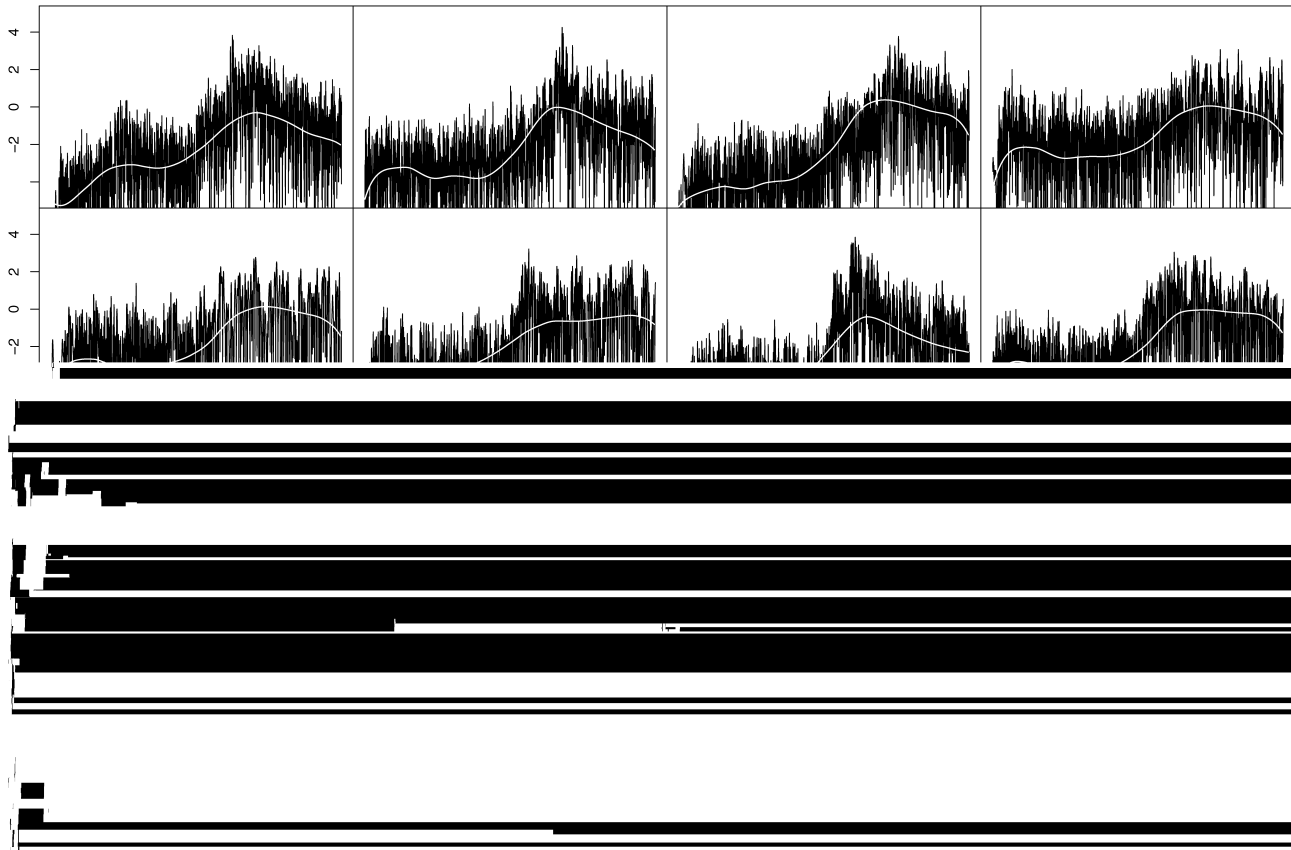
3. ESTIMATION OF MODEL COMPONENTS

The maximum likelihood estimates of the parameters of the model are given by (2005) (1998).

$$Z_{ij} = \begin{cases} X_{ij} - S_i(t_{ij}) & \text{if } i = 1, \dots, n, j = 1, \dots, m, \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The maximum likelihood estimates of the parameters of the model are given by (2005) (1998).

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$F_i = 3.0$... $E_i = E$... $F_i = 1.0$... $E_i = 0.025$...

(.6). ... μ_V ... $G_V(s, t)$... $Q_V(t)$...

$$V_i(t) = \mu_V(t) + \sum_{k=1}^M \lambda_{ik} \phi_k(t). \quad (13)$$

3.

4. A MP O IC RE L

(13)

Z_{ij} (3) ... S_i ... b_S ... $b_{S,i}$...

Theorem 1. ... $S_i(t)$...

$$E_{i \in \mathcal{I}} \left(S_i(t) \mid S_i(t) \right) = O \left(b_S^2 + \frac{1}{mb_S} \right). \quad (14)$$

$\mu_V(t) = \int_{k \in T} \mu_V(t, k) G_V(s, t) dk$
 (2), (3), (4)
 $\frac{2}{W} (2)$

Theorem 2. (1) (8), (1.1) (2.2),

$$\begin{aligned}
 & \int_{t \in T} \mu_V(t) \mu_V(t) \\
 &= O_p \left(b_S^2 + \frac{1}{mb_S} + \frac{1}{nb_V} \right), \\
 & \int_{s, t \in T} G_V(s, t) G_V(s, t) \\
 &= O_p \left(b_S^2 + \frac{1}{mb_S} + \frac{1}{nh_V^2} \right), \\
 & \frac{2}{W} \frac{2}{W} \\
 &= O_p \left(b_S^2 + \frac{1}{mb_S} + \frac{1}{nh_V^2} + \frac{1}{nb_{Q_V}} \right).
 \end{aligned} \tag{15}$$

$k \in T, k \in T, 1, k \in T$
 $\int_{t \in T} k(t) k(t) \xrightarrow{p} 0, k \xrightarrow{p} k.$ (16)

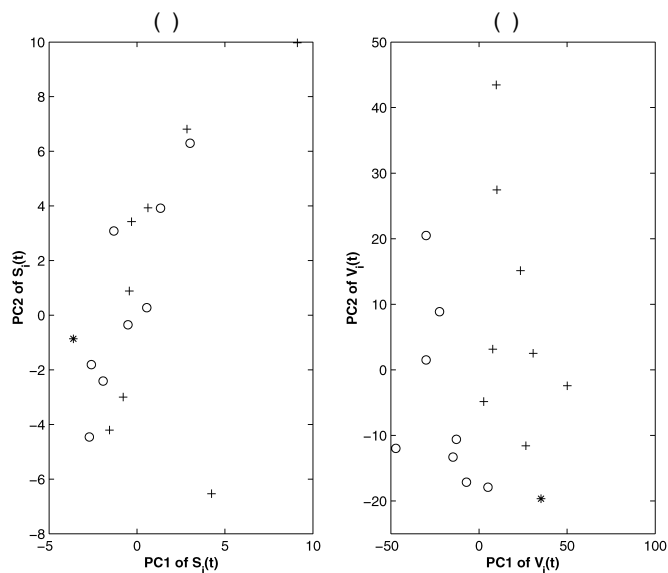
$k(t) = O_p \left(nk + \frac{1}{nk} \right), k = O_p \left(nk + \frac{1}{nk} \right),$
 $nk \frac{1}{nk} \frac{1}{nk} (1)$
 (1.1) (2.2)

V_i (13)
 $V_i, 3,$
 $ik \frac{1}{V_i} (5)$

Theorem 3. (1) (8), (1.1) (2.2),

V
 $\int_{1 \leq k \leq M} ik ik \xrightarrow{p} 0,$ (17)
 (13), $M = M(n) \rightarrow \infty, n \rightarrow \infty,$
 $\int_{1 \leq i \leq n} V_i(t) V_i(t) \xrightarrow{p} 0.$ (18)
 (16),

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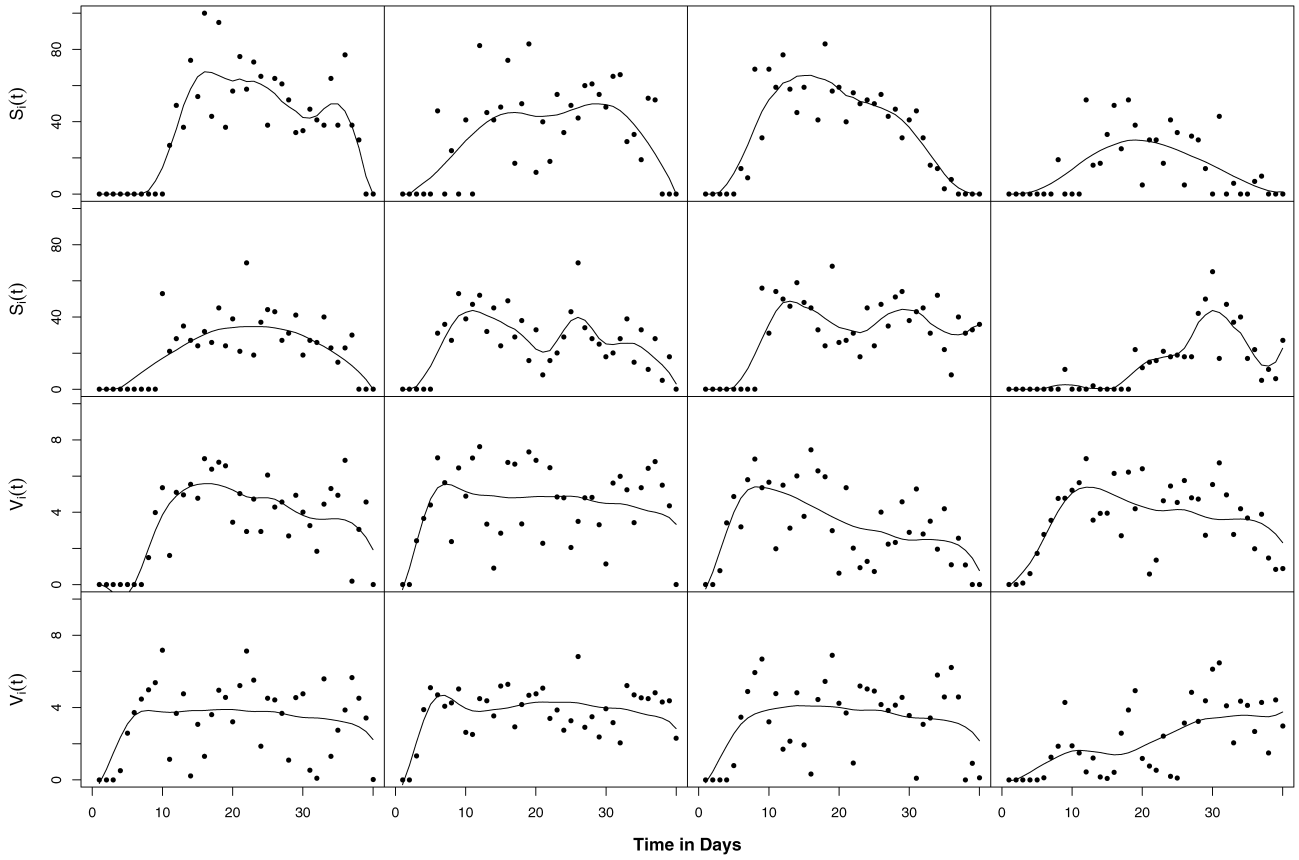
$i, k = 1, 2, i = 1, \dots, 15$ (5), $V_i(t)$

5.2 Error-Limit D

$2.2(1) - 0.1$

$F_i, C_n, R, PC2, F_i, E, P_i, P, F, n, E, D, (+, *,)$

$(X_{ij} - S_i(t_{ij}))^2, 0, .001, R_{ij}^2, V_i, 3, 1, ik, k = 1, 2, i = 1, \dots, 15, (8), S(\mathbb{R}), 1991, 4(2), 2003, V_i, S_i, V_i, S, 4(2), S, (i_1, i_2), S_i, 7, (15), 0, S,$



$F_i = 5.0$, $E_i = L$, $C = ()$, $n = 1$, $E_i = M$, $F_i = 40D$, $A_i = B$, $C = L$, $O = O$, $C = ()$, $E = P$, $O = ()$, $R^2 = (R^2(\hat{\beta}))$, $E = m$, $n = R$, $n = (13)$, $F = P$, $n = E_i = M$, (n) .

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APPENDI A: E IMA ION PROCED RE

1() 2() (1.2.1) (1.2.2)
 $b_V = b_V(n)$ $h_V = h_V(n)$ μ_V
 (4) G_V (5) 1 2 Z_{ij}

(1996)
 $S_i, i = 1, \dots, n, (t_{ij}, X_{ij}),$
 $j = 1, \dots, m, b_{S,i}$

$$\sum_{j=1}^m 1\left(\frac{t_{ij} - t}{b_{S,i}}\right) \{X_{ij} - i,0 - i,1(t - t_{ij})\}^2 \quad (1)$$

$i,0 - i,1$ $S_i(t_{ij}) =$
 $i,0(t_{ij})$ $b_{S,i}$ (2.1)

$$\sum_{i=1}^n \sum_{j=1}^m 1\left(\frac{t_{ij} - t}{b_V}\right) \{Z_{ij} - 0 - 1(t - t_{ij})\}^2 \quad (2)$$

$0 - 1$ $\mu_V(t) = 0(t)$ $i(t_{j1}, t_{j2}) =$
 $(Z_i(t_{j1}) - \mu_V(t_{j1}))(Z_i(t_{j2}) - \mu_V(t_{j2}))$
 $G_V(s, t)$

$$\sum_{i=1}^n \sum_{j_1 \neq j_2}^m 2\left(\frac{t_{j_1} - s}{h_V}, \frac{t_{j_2} - t}{h_V}\right) \{i(t_{j_1}, t_{j_2}) - f(\cdot, (s, t), (t_{j_1}, t_{j_2}))\}^2 \quad (3)$$

$f(\cdot, (s, t), (t_{j_1}, t_{j_2})) = 0 + 11(s - t_{j_1}) + 12(t - t_{j_2}),$
 $(0, 11, 12), G_V(s, t) = 0(s, t).$
 $\{k, k\}k$ $\{k, k\}k$

$$\int_T G_V(s, t) k(s) ds = k k(t), \quad (4)$$

$\{k\}k$ 1 (2003).
 M ik (8),

$$ik = \sum_{j=2}^m (Z_{ij} - \mu_V(t_{ij})) k(t_{ij})(t_{ij} - t_{i,j-1}), \quad i = 1, \dots, n, k = 1, \dots, M. \quad (5)$$

$$\mu_V^{(i)}(t) = \sum_{k=1}^M \mu_{ik}^{(i)}(t) \quad (i)$$

$$V(M) = \sum_{i=1}^n \sum_{j=1}^m \{Z_{ij} - V_i^{(i)}(t_{ij})\}^2, \quad (6)$$

$$V_i^{(i)}(t) = \mu_V^{(i)}(t) + \sum_{k=1}^M \mu_{ik}^{(i)}(t) \quad (i)$$

$$(5) \dots (13).$$

$\frac{2}{W}$
 G_V
 $G_V^*(t)$
 $\{G_V(t, t) + \frac{2}{W}\} Q_V(s)$
 b_{QV} $T = a_1, a_2$ $T = a_2 - a_1$ $T_1 = a_1 + T/4, a_2 - T/4$

$$\frac{2}{W} = \frac{1}{T_1} \int_{T_1} \{Q_V(t) - G_V^*(t)\} dt \quad (7)$$

$\frac{2}{W} > 0$ $\frac{2}{W} = 0$ $T/4$ (2003).

APPENDI B: A MP ION AND NO A ION

S V

$$(1) \dots C > 0$$

$$|S^{(l)}(t)| < C \quad l = 0, 1, 2$$

$$|V(t)| < C.$$

$b_{S,i} = b_{S,i}(n)$ $b_V = b_V(n)$ $h_V = h_V(n)$
 $b_{QV} = b_{QV}(n)$ S_i (1) μ_V
 (2) G_V (3) $Q_V(t)$ (7)

$$(2.1) \dots b_{S,i} \dots c_1 \dots c_2, \quad 0 < c_1 < \dots b_{S,i}/b_S \dots b_{S,i}/b_S < c_2 < \infty.$$

$$(2.2) m \rightarrow \infty, b_S \rightarrow 0, \dots mb_S^2 \rightarrow \infty.$$

$$(2.3) b_V \rightarrow 0, b_{QV} \rightarrow 0, nb_V^4 \rightarrow \infty, nb_{QV}^4 \rightarrow \infty, \dots nb_V^6 < \infty, \dots nb_{QV}^6 < \infty.$$

$$(2.4) h_V \rightarrow 0, nh_V^6 \rightarrow \infty, \dots nh_V^8 < \infty.$$

$$(2.5) \dots n^{1/2} b_V m^{-1} < \infty, \dots n^{1/2} b_{QV} m^{-1} < \infty, \dots n^{1/2} h_V m^{-1} < \infty.$$

$\{t_{ij}\}_{i=1, \dots, n, j=1, \dots, m}$
 $i, j = 1, \dots, m - 1, t_{ij} < t_{i,j+1}$
 $\int_T f(t) dt = 1, \dots t \in T, f(t) > 0$
 $t_{ij} \dots t_{ij} = F^{-1}\left(\frac{j-1}{m-1}\right), \dots F^{-1}$
 $F(t) = \int_{a_1}^t f(s) ds$
 $c_1 \dots c_2$
 $0 < c_1 < \dots i \dots t \in T, f_i(t) < c_2$
 N_i
 $m \rightarrow \infty$
 $0 < c_1 < \dots \frac{N_i}{m} < \dots \frac{N_i}{m} < c_2 < \infty$
 $N_i = m$

..., $m\}$, $n = \lfloor \sum_{j=1}^m \{t_{ij} - t_{i,j-1}\} : j = 2, \dots, m\}$, $n, m \rightarrow \infty$.

$$(3) \quad n = O(m^{-1}), \quad n, m \rightarrow \infty.$$

$X_{ij}, Z_{ij}, t \in \mathcal{T}$, X_{ij}, Z_{ij}

$$(4) \quad \sum_{j=1}^m E X_{ij}^4 < \infty, \quad \sum_{j=1}^m E Z_{ij}^4 < \infty.$$

H (1989). $(f \cdot g)(h) = \{f, h\} g$, $f, g, h \in H$.

H $F = (T_1, T_2) = \sum_{j=1}^m \{T_{1j} u_j, T_{2j} u_j\}$, $T_F = (T, T_F)$, $T_1, T_2, T \in F, T_2 = \{u_j : j = 1, \dots, m\}$.

$$(6) \quad G_V = \sum_{k \in \mathcal{I}_j} k, \quad G_V = \sum_{k \in \mathcal{I}_j} k, \quad (5)$$

$G_V = \sum_{k \in \mathcal{I}_j} k, \quad G_V = \sum_{k \in \mathcal{I}_j} k$

$\mathcal{I}_i = \{j : t_{ij} = 1\}$, $\mathcal{I}_j = \{i : t_{ij} = 1\}$, $\mathcal{I}_i = \{j : t_{ij} = 1\}$, $\mathcal{I}_j = \{i : t_{ij} = 1\}$.

$$y_j = \frac{1}{2} \sum_{i \in \mathcal{I}_j} \{1 - t_{ij}\}, \quad (1.1)$$

$$\Lambda y_j = \{z \in \mathcal{C} : z_j = y_j\}, \quad \mathcal{C}$$

$$R_V(z) = (G_V - zI)^{-1} R_V(z) = (G_V - zI)^{-1} R_V(z)$$

$$A y_j = \{R_V(z) : z \in \Lambda y_j\} \quad (1.2)$$

$M = M(n)$

$$V_i(t) = \mu_V(t) + \sum_{m=1}^{M(n)} \{i_m - m(t)\} \quad (13)$$

$M = M(n)$, $n, m \rightarrow \infty$.

$$(5) \quad n = \sum_{j=1}^m (V_j A y_j - j \infty) / (\bar{n} h_V^2 A y_j) \rightarrow 0, \quad M = M(n) \rightarrow \infty;$$

$$(6) \quad \sum_{j=1}^m j \infty = o(\bar{n} b_V, \bar{m}), \quad \sum_{j=1}^m j \infty = o(m).$$

(5), (6)

$m = b_S^2 + (\bar{m} b_S)^{-1}$, V

$$(7) \quad E \{ \sum_{t \in \mathcal{T}} V(t) - V^{(M)}(t) \}^2 = o(n), \quad V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^M k k(t).$$

$$(8) \quad \sum_{k=1}^M (V_k A y_k - k \infty) / (m^{-1} A y_k) \rightarrow 0, \quad n \rightarrow \infty.$$

$t = t_{ij}, t_1 = t_{ij_1}, t_2 = t_{ij_2}, i, j, j_1, j_2$, $g(x, t) = X_{ij}, g_2(x_1, x_2, t_1, t_2) = (X_{ij_1}, X_{ij_2}), f(z, t) = Z_{ij}, f_2(z_1, z_2, t_1, t_2) = (Z_{ij_1}, Z_{ij_2}), g(t), f(t), t \in \mathcal{T}, g_2(t_1, t_2), f_2(t_1, t_2), t_1, t_2 \in \mathcal{T}$.

$$(1.1) \quad (d^2/dt^2)g(x, t) = (d^2/dt^2)f(z, t), \quad \mathbb{R} \times \mathcal{T}$$

$$(1.2) \quad (d^2/dt_1^1 dt_2^2)g_2(x_1, x_2, t_1, t_2) = (d^2/dt_1^1 dt_2^2)f_2(z_1, z_2, t_1, t_2), \quad \mathbb{R}^2 \times \mathcal{T}^2, \quad 1+2=2, 0 \leq 1, 2 \leq 2.$$

$\mathbb{R} \rightarrow \mathbb{R}, \mathbb{R}^2 \rightarrow \mathbb{R}$, $\|f(t)\| = \int e^{iut} f_1(u) du, \|f_2(t, s)\| = \int e^{i(ut+ivs)} f_2(u, v) dudv$.

$$(1.1) \quad \int_1^2 f_1^2(u) du < \infty, \quad \int_1^2 f_1(t) dt < \infty.$$

$$(1.2) \quad \int_1^2 \int_1^2 f_2^2(u, v) dudv < \infty, \quad \int_1^2 \int_1^2 f_2(t, s) dt ds < \infty.$$

APPENDIX C: PROOF

$P_n = 1$, $W = S_V, E^* = V$

$R_j = R_j(1)$, $R_{ij} = (1)$, $E^*(R_j) = 0$, $E^*(R_j^2) < C_1$

$C_1 = 3$, (1) , (1) , (1979)

$$E^* \left(\sum_{t \in \mathcal{T}} S(t) - S(t) \right) (1) = O \left(b_S^2 + \frac{1}{mb_S} \right) (1).$$

$S^{(j)}(1), j = 0, 1, 2, V(1)$, (1) , (1)

$$E^* \left(\sum_{t \in \mathcal{T}} S(t) - S(t) \right) (1) = O \left(b_S^2 + \frac{1}{mb_S} \right),$$

(14).

$\{t_{ij}, Z_{ij}\}$, $\mu_V, G_V, \frac{2}{W}, k, k, ik$, (2), (3), (4), (5)

$$nk = \frac{V_k A y_k}{\bar{n} h_V^2 A y_k}, \quad nk^* = \frac{V_k A y_k}{m^{-1} A y_k}, \quad (1.1)$$

$m = b_S^2 + (\bar{m} b_S)^{-1}, V_k, A y_k$, (1.1), (1.2)

Lemma C.1. (2.1) (2.3), (3) (5), (1.1) (1.2)

$$\sum_{t \in \mathcal{T}} \mu_V(t) - \mu_V(t) = O_p \left(\frac{1}{\bar{n} b_V} \right), \quad (1.2)$$

$$\sum_{s, t \in \mathcal{T}} G_V(s, t) - G_V(s, t) = O_p \left(\frac{1}{\bar{n} h_V^2} \right).$$

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$$\sum_{t \in \mathcal{T}} \dots k(t) = O_p(nk) \quad (4.3)$$

$$\dots k = O_p(nk),$$

Since $nk \rightarrow 0$ as $n \rightarrow \infty$, $k \dots nk \dots (4.1)$, $O_p(\dots) \dots (4.3)$ $\dots k = M \dots (4.2)$,

$$\sum_{t \in \mathcal{T}} \frac{2}{W}(t) = O_p\left(\left\{\frac{1}{nh_V^2}, \frac{1}{nb_{Q_V}}\right\}\right). \quad (4.4)$$

By (1) (7), (4.1) (4.2),

$$\sum_{k=1}^M ik \dots \xrightarrow{p} 0 \quad (4.5)$$

$$\sum_{t \in \mathcal{T}} \left| \sum_{k=1}^M ik \dots k(t) - \sum_{k=1}^{\infty} ik \dots k(t) \right| \xrightarrow{p} 0,$$

Since $M \dots M = M(n) \rightarrow \infty$ as $n \rightarrow \infty$.

Proof of Lemma C.1. By (4.2), (4.3), (4.5) \dots (2006) $\dots W$

$$+ \left| \sum_{t \in \mathcal{T}} \left\{ \sum_{k=1}^M i_k \cdot k(t) - \sum_{k=1}^{\infty} i_k \cdot k(t) \right\} \right|$$

$$Q_{i1}(n) + Q_{i2}(n),$$

where $Q_{i1}(n) \xrightarrow{p} 0$, $Q_{i2}(n) \xrightarrow{p} 0$.
 (5), $Q_{i2}(n) \xrightarrow{p} 0$, (1) (7),
 $Q_{i2}(n) = O_p(\frac{1}{n})$, $O_p(\frac{1}{n})$
 $Q_{i1}(n)$,

$$Q_{i1}(n) = \left| \sum_{t \in \mathcal{T}} \left\{ \sum_{k=1}^M i_k \cdot k(t) - \sum_{k=1}^{\infty} i_k \cdot k(t) \right\} \right| \quad (11)$$

(10), (11)

$$C_1 \sum_{k=1}^M k^2 + n \left\{ C_2 + \sum_{j=2}^m Z_{ij}(t_{ij} - t_{i,j-1}) \right\} \xrightarrow{p} 0.$$

(11)
 $O_p(\sum_{k=1}^M \frac{1}{k} \sqrt{E i_k} / (m^1 A_k))$, $E(\sum_{k=1}^M \frac{1}{k} \sqrt{E i_k} / (m^1 A_k)) \rightarrow 0$,
 $Q_{i1}(n) = O_p(\frac{1}{n})$, $O_p(\frac{1}{n})$
 (18) $\sum_{t \in \mathcal{T}} V_i(t) - V_i(t) = O_p(\frac{1}{n} + \frac{1}{n})$,
 $O_p(\frac{1}{n})$

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