H. -G M LLER, AD M LLER, F. AO

1. IN ROD C ION

1974), (1999), 1995), (1986), 1987), 1993). (1998), (1998), (2000), (2004), g,

$$Y_j = g(t_j) + e_j(t_j), \qquad j = 1, \ldots, J.$$

$(t_j)_{j=1,\ldots,J}$ $v(t_j) =$ $(e_j(t_j))$ (X,Y), $v(x) = E(Y^2 \; X = x) \quad E(Y \; X = x) \; 2$.

(2002, 2005). (2002) (1998). 1. (2001) 1

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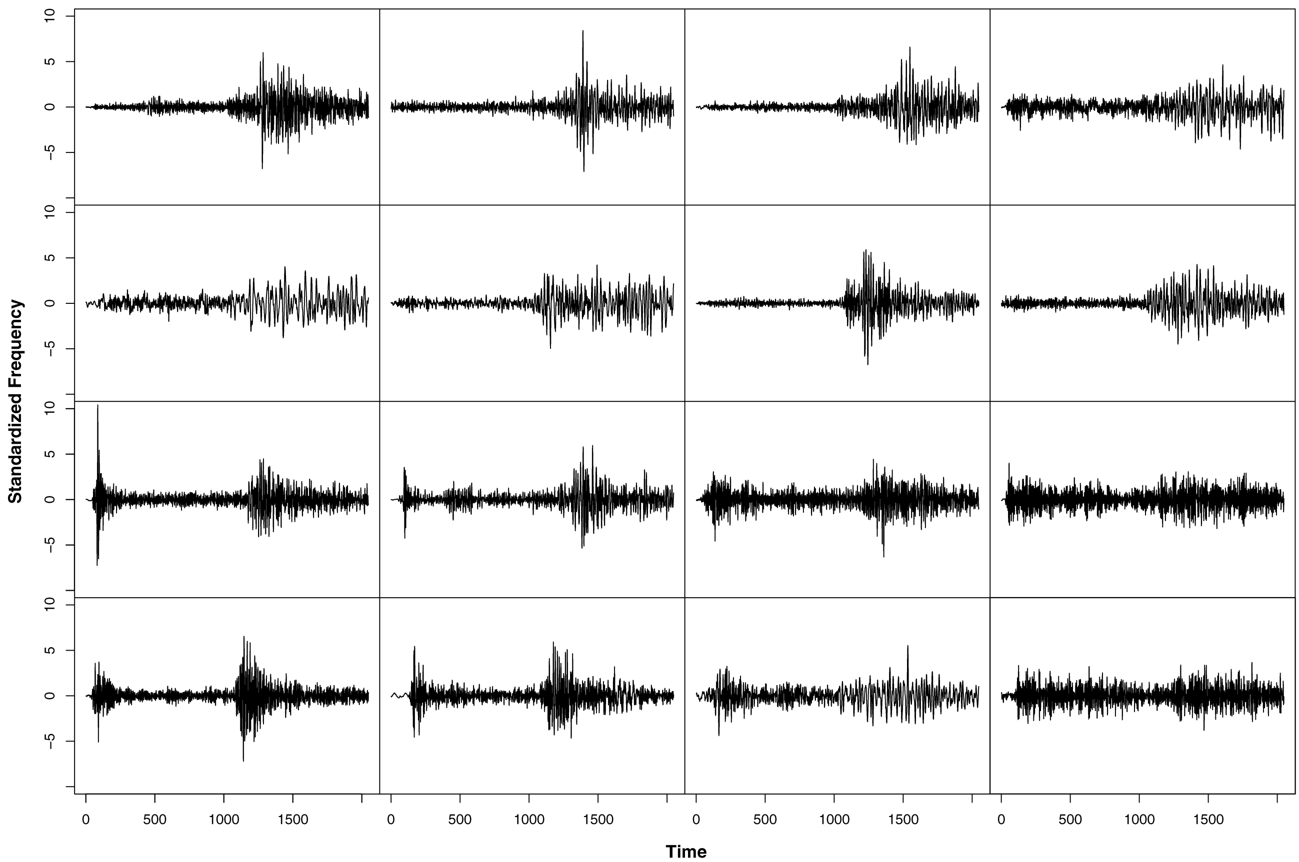


F 1. D. E ... E ( ), ... E (...), ... O. E ( ). ... .025

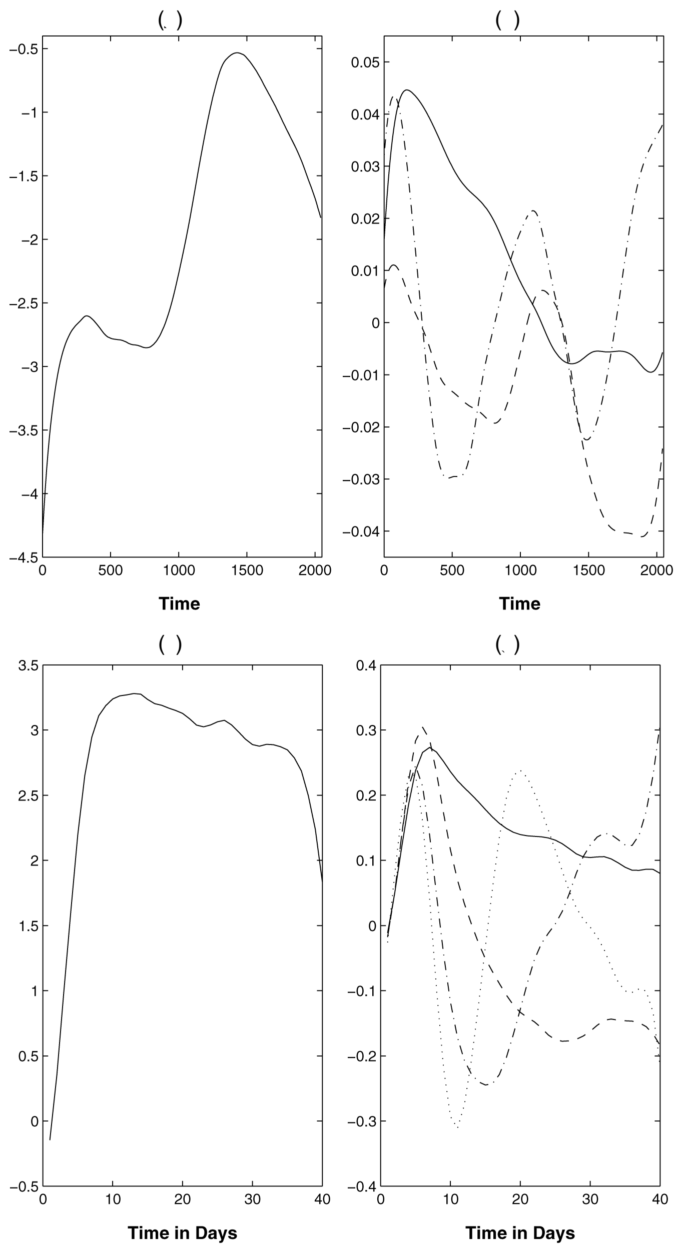
... 1. ... 2. ... 1986). ... (1950) ... (1958), ... (... 1991; ... 1992; ... 2005; ... 2001; ... 2003).

... 3, ... 4 ... (. 1), ... (. 2), ... (. 3) ... 5, ...

2. DECOMPO ING F NC IONAL DA A

S ... V ...

... S ... n ... S_i, $i = 1, \ldots, n$. ... X_{ij} ...



F 2. C n F P E E D ( ) ( ) E -L D ( ) ( ) ( ) ( ) n ( ) n n ( ) ( ). ( ) n n ( ), (----), (- - - -) ( ) 62.8%, 23.6%, 7.8% ( ) n n ( ), (----), (- - - -), ( ) ( ) 48.3%, 21.0%, 11.6%, 6.7% n .025 ( ) ( ), ( ) ( ).

t_{ij} $=$ a_1, a_2 S

$$X_{ij} = S_i(t_{ij}) + R_{ij}, \qquad i = 1, \dots, n, j = 1, \dots, m. \tag{1}$$

R_{ij} , R_{ij} $R_{i'k}$ $i \neq i'$,

$$ER_{ij} = 0, \qquad (R_{ij}) = \sigma^2_{Rij} < \infty.$$

R_{ij} i S $= a_1, a_2$ m (2.5). R_{ij}

, (2)

$$(Z_{ij}, Z_{ik}) = \big(V_i(t_{ij}), V_i(t_{ik})\big) = G_V(t_{ij}, t_{ik}), \qquad j \neq k. \tag{9}$$

G_V Z_{ij}. σ_W^2. (1998) , (2005). μ_V G_V (1991).

, φ_k ψ_k, $k = 1, 2, \ldots,$ λ_k ρ_k $\sum_k \lambda_k < \infty$ $\sum_k \rho_k < \infty$. ξ_k $E(\xi_k) = 0$ $(\xi_k) = \lambda_k$ ζ_k $E(\zeta_k) = 0$ $(\zeta_k) = \rho_k$, μ_S μ_V ,

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} \xi_k \varphi_k(t)$$
$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} \zeta_k \psi_k(t). \tag{10}$$

W_{ij} $\varepsilon_{ij}, i = 1, \ldots, n,\ j = 1, \ldots, m,$ $E(W_{ij}) = 0$, $(W_{ij}) = \sigma_W^2$, $P(\varepsilon_{ij} > 0) = P(\varepsilon_{ij} < 0) = \frac{1}{2}$. X_{ij}

$$X_{ij} = S_i(t_{ij}) + (\varepsilon_{ij})\{ V_i(t_{ij}) + W_{ij} \}^{1/2}. \tag{11}$$

S V S, λ_k ρ_k 0 ξ_k ζ_k.

3. E IMA ION OF MODEL COMPONEN

Z_{ij} V_i (7), ζ_k V, (8). . (2003), $V(t_{ij})$ Z_{ij} μ_V ψ_k μ_k ψ_k. $(W_{ij}) = \sigma_W^2$.

X_{ij} $(t_{ij}), i = 1, \ldots, n, j = 1, \ldots, m,$ i j (1), (7) ( ). (2005) (1998).

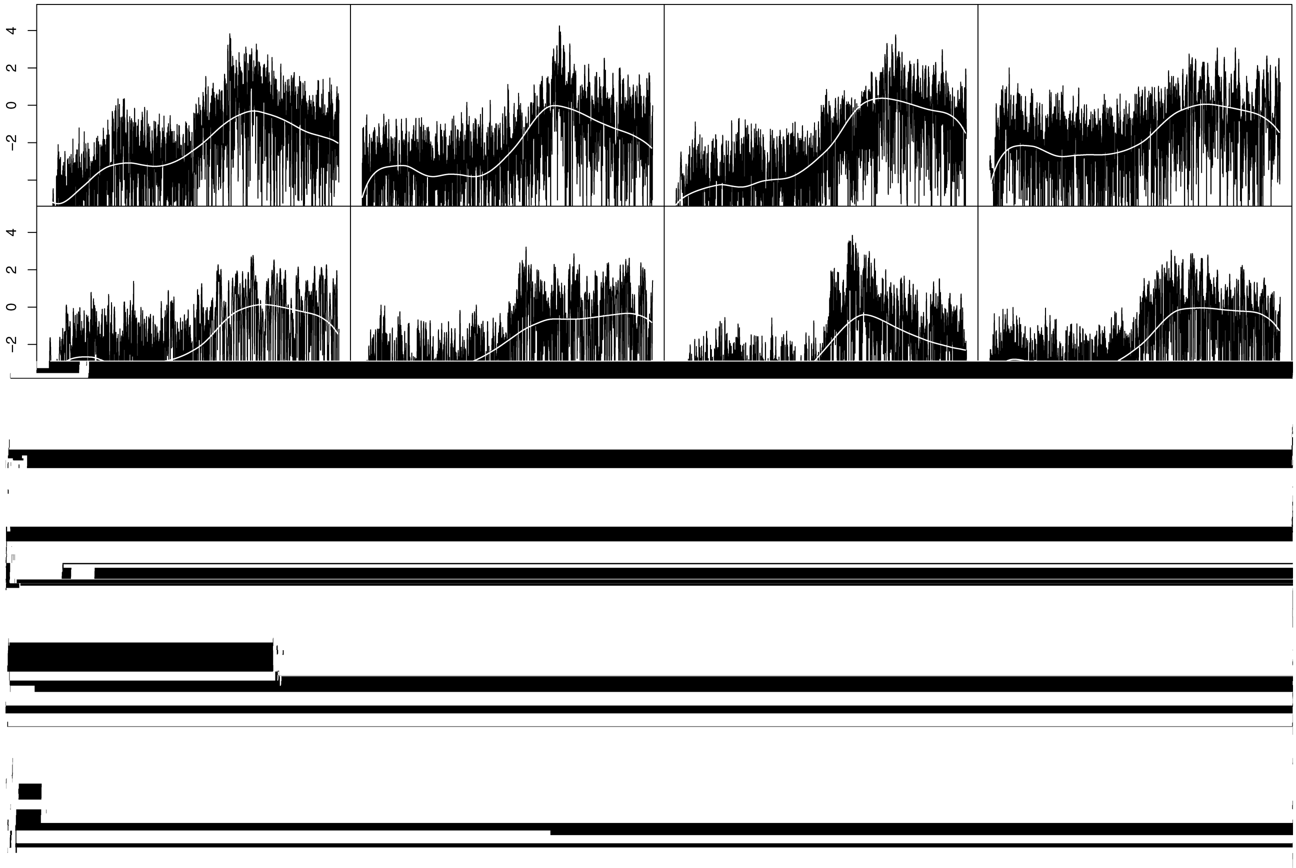
; . $(t_{ij}, X_{ij}),\ j = 1, \ldots, m,$ S_i; $b_{S,i}$; $b_{S,i}$ S. $S_i(t_{ij})$ (.1) $S_i(t_{ij})$. $R_{ij} = X_{ij}\quad S_i(t_{ij})$

$$Z_{ij} = (R_{ij}^2) = \big(X_{ij}\quad S_i(t_{ij})\big)^2, \qquad i = 1, \ldots, n, j = 1, \ldots, m. \tag{12}$$

$Z_{ij}, i = 1, \ldots, n, j = 1, \ldots, m,$:

1. Z_{ij}, μ_V (4) b_V ; (.2) b_V
2. G_V (5) (.3) , h_V, $(t_{ij}, t_{ij'})$, $t_{ij} \neq t_{ij'}$, σ_W^2; (2).
3. , ; (.4).
4. $(W_{ij}) = \sigma_W^2$ (2) . b_{Q_W} σ_W^2 (.7). , b_{Q_W}
5. ζ_j (8) (.5).

M V, ;



F 3. O = $(R^2(\;))$ ( ) E F P ( ) (13) ( ) E E ( ), E ( ), E ( ). ; F 1. .025

(.6). (2005),

μ_V, ψ_k ρ_k, σ_W^2, ζ_{ik}. (7), M

$$V_i(t) = \mu_V(t) + \sum_{k=1}^{M} \zeta_{ik}\psi_k(t). \tag{13}$$

3.

4. A MP O IC RE L

(13) Z_{ij} (3) Z_{ij} (12) S_i. b_S S_i (.1) ; b_S $b_{S,i}$,

$b_{S,i}$, b_S, (2.1), b_S, (2.2); b_V, h_V, b_{Q_V} μ_V (.2), $G_V(s,t)$ (.3), $Q_V(t)$ (.7) (2.3), (2.5).

m ( ) ( )

Theorem 1. (1), (2), (1.1), (2.1), $S_i(t)$

$$E\Big(\sup_{t\in\mathcal{T}} S_i(t) \quad S_i(t)\Big) = O\Big(b_S^2 + \frac{1}{\overline{m}b_S}\Big). \tag{14}$$

47.7( )0. 2( )1(-25 247.731 5 1 13.5653 02

$\mu_V(t)$ $\mu_V(t)$, $G_V(s,t)$ $G_V(s,t)$, ρ_k $\psi_k(t)$ ρ_k ψ_k. (.2), (.3), (.4) σ_W^2 (.7) σ_W^2 (2) .

Theorem 2. (1), (8) (1.1), (2.2),

$$\begin{aligned}
&\mu_V(t) \quad \mu_V(t) \\
&\quad = O_p\left(b_S^2 + \frac{1}{\overline{m}b_S} + \frac{1}{\overline{n}b_V}\right), \\
&G_V(s,t) \quad G_V(s,t) \\
&\quad = O_p\left(b_S^2 + \frac{1}{\overline{m}b_S} + \frac{1}{\overline{n}h_V^2}\right), \\
&\sigma_W^2 \quad \sigma_W^2 \\
&\quad = O_p\left(b_S^2 + \frac{1}{\overline{m}b_S} + \frac{1}{\overline{n}h_V^2} + \frac{1}{\overline{n}b_{Q_V}}\right).
\end{aligned} \tag{15}$$

ρ_k 1, ψ_k

$$\psi_k(t) \quad \psi_k(t) \xrightarrow{p} 0, \qquad \rho_k \xrightarrow{p} \rho_k. \tag{16}$$

ρ_k ψ_k $t\in\mathcal{T}$ $\psi_k(t)$ $\psi_k(t) = O_p(\alpha_{nk} + \alpha_{nk})$ ρ_k $\rho_k = O_p(\alpha_{nk} + \alpha_{nk})$, α_{nk} α_{nk} (.1) (.1) (.2).

V_i (13) V_i, 3. ζ_{ik} (.5) ζ_{ik} V.

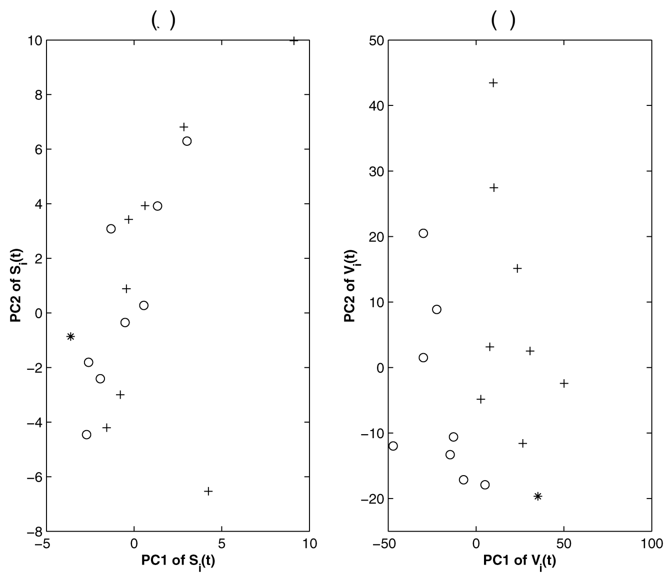
Theorem 3. (1), (8) (1.1), (2.2), V

$$\zeta_{ik} \quad \zeta_{ik} \xrightarrow{p} 0, \tag{17}$$

(13), $M = M(n) \to \infty$, $n \to \infty$. $V_i(t)$ V, 1 i n,

$$V_i(t) \quad V_i(t) \xrightarrow{p} 0. \tag{18}$$

(16),



F 4. R F E F P C , PC2 PC1, O P ( ) F P ( ), E E D (+, ; ;).

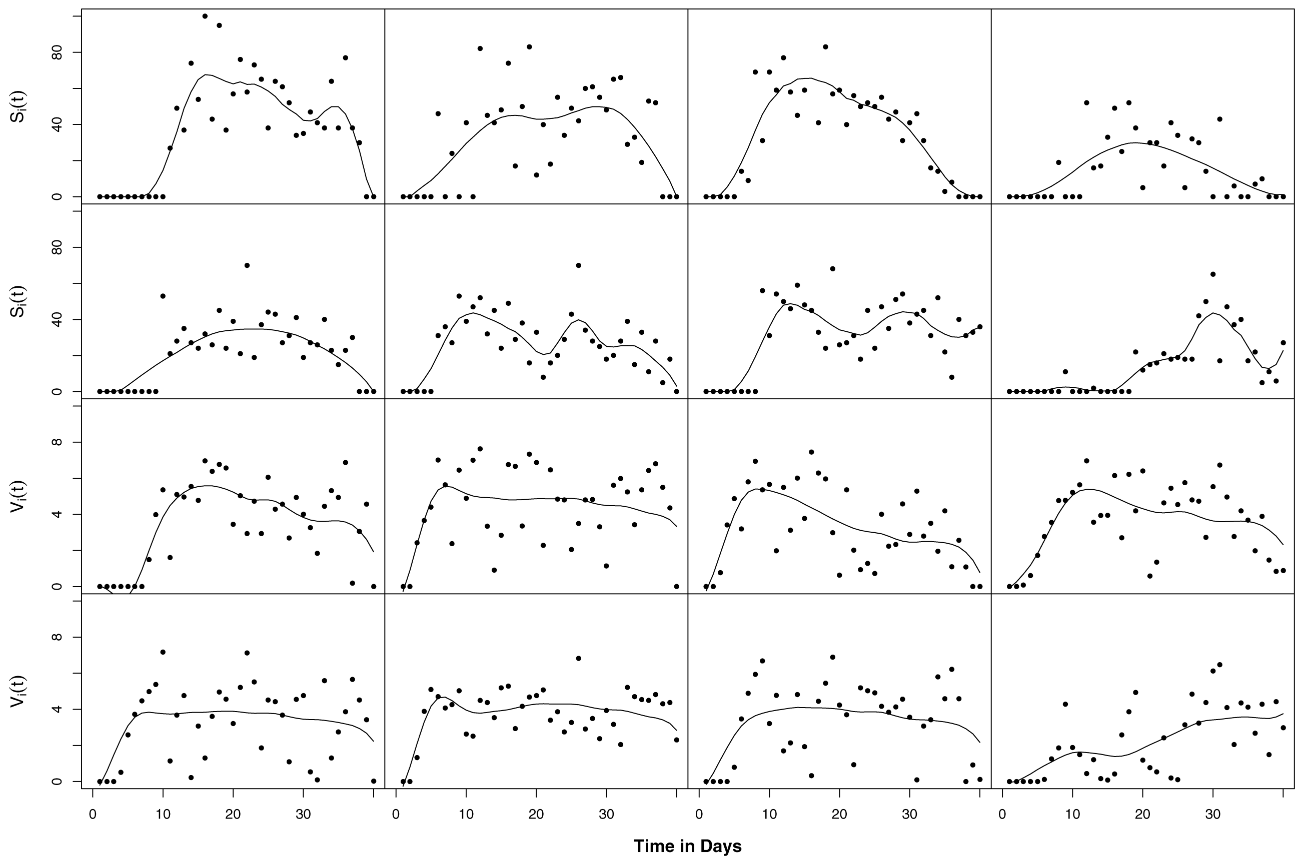
$(X_{ij}\ \ S_i(t_{ij}))^2$ 0, .001 R^2_{ij} .

V_i 1 3. 1.

ζ_{ik}, $k = 1, 2$, $i = 1, \ldots, 15$ (.5) , (8) V, 4( ), S (1991, . 2003) 4( ). V_i S_i. V_i S, 4( ), S, (ζ_{i1}, ζ_{i2}) ; , S 7 (15), 0 . S,

ζ_{ik}, $k = 1, 2$, $V_i(t)$. , .

5.2 E -L D



$S_i(t)$
$V_i(t)$
Time in Days

APPENDI A: E IMA ION PROCED RE

$\kappa_1(\,)$ $\kappa_2(\,,\,)$ (2.1) (2.2). $b_V = b_V(n)$ $h_V = h_V(n)$ μ_V (4) G_V (5) 1 2 Z_{ij}.

(1996) S_i, $i = 1, \ldots, n$, (t_{ij}, X_{ij}), $j = 1, \ldots, m$, $b_{S,i}$

$$\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} \quad t}{b_{S,i}}\right) X_{ij} \quad \beta_{i,0} \quad \beta_{i,1}(t \quad t_{ij})\ ^2 \qquad (.1)$$

$\beta_{i,0}$ $\beta_{i,1}$. $S_i(t_{ij}) = \beta_{i,0}(t_{ij})$. $b_{S,i}$ b_S (2.1). μ_V,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij} \quad t}{b_V}\right)\{Z_{ij} \quad \beta_0 \quad \beta_1(t \quad t_{ij})\}^2 \qquad (.2)$$

β_0 β_1, $\mu_V(t) = \beta_0(t)$. $_i(t_{ij_1}, t_{ij_2}) = (Z_i(t_{ij_1}) \quad \mu_V(t_{ij_1}))(Z_i(t_{ij_2}) \quad \mu_V(t_{ij_2}))$, $G_V(s,t)$

$$\sum_{i=1}^{n} \ 1 \sum_{j_1 \neq j_2 \quad m} \kappa_2\left(\frac{t_{ij_1} \quad s}{h_V}, \frac{t_{ij_2} \quad t}{h_V}\right)$$

$$\{ \ _i(t_{ij_1}, t_{ij_2}) \quad f(\beta, (s,t), (t_{ij_1}, t_{ij_2}))\}^2, \qquad (.3)$$

$f(\beta, (s,t), (t_{ij_1}, t_{ij_2})) = \beta_0 + \beta_{11}(s \quad t_{ij_1}) + \beta_{12}(t \quad t_{ij_2})$, $\beta = (\beta_0, \beta_{11}, \beta_{12})$, $G_V(s,t) = \beta_0(s,t)$.

$\rho_k, \psi_k \ k \ 1$ ρ_k, $\psi_k \ k \ 1$

$$\int_{\mathcal{T}} G_V(s,t)\psi_k(s)\,ds = \rho_k\psi_k(t), \qquad (.4)$$

$\psi_k \ k \ 1$ (, . 2003). M ζ_{ik} (8),

$$\zeta_{ik} = \sum_{j=2}^{m} (Z_{ij} \quad \mu_V(t_{ij}))\psi_k(t_{ij})(t_{ij} \quad t_{i,j \ 1}),$$

$$i = 1, \ldots, n, k = 1, \ldots, M. \qquad (.5)$$

$\mu_V^{(\ i)}$ $\psi_k^{(\ i)}$ i

$$_V(M) = \sum_{i=1}^{n}\sum_{j=1}^{m}\{Z_{ij} \quad V_i^{(\ i)}(t_{ij})\}^2, \qquad (.6)$$

M, $V_i^{(\ i)}(t) = \mu_V^{(\ i)}(t) + \sum_{k=1}^{M} \zeta_{ik}^{(\ i)}\psi_k^{(\ i)}(t)$ $\zeta_{ik}^{(\ i)}$ (.5). V_i (13).

σ_W^2, G_V, $G_V(t)$ $G_V(t,t) + \sigma_W^2$ $Q_V(s)$, b_{Q_V}. $\mathcal{T} = a_1, a_2$, $\mathcal{T} = a_2 \quad a_1$ $\mathcal{T}_1 = a_1 + \mathcal{T}/4, a_2 \quad \mathcal{T}/4$.

$$\sigma_W^2 = \frac{1}{\mathcal{T}_1}\int_{\mathcal{T}_1} Q_V(t) \quad G_V(t)\ _+ dt \qquad (.7)$$

$\sigma_W^2 > 0$ $\sigma_W^2 = 0$ $\mathcal{T}/4$ (. 2003).

APPENDI B: A MP ION AND NO A ION

S V :

(1) $C > 0$ S V

$$_t \left|S^{(\nu)}(t)\right| < C \qquad \nu = 0, 1, 2$$

$$_t \ V(t) < C.$$

$b_{S,i} = b_{S,i}(n)$, $b_V = b_V(n)$, $h_V = h_V(n)$, $b_{Q_V} = b_{Q_V}(n)$ S_i (.1), μ_V (.2), G_V (.3), $Q_V(t)$ (.7). n m :

(2.1) $b_{S,i}$, b_S c_1 c_2, $0 < c_1 < \ _i b_{S,i}/b_S \ _i b_{S,i}/b_S < c_2 < \infty$.

(2.2) $m \to \infty$, $b_S \to 0$, $mb_S^2 \to \infty$.

(2.3) $b_V \to 0$, $b_{Q_V} \to 0$, $nb_V^4 \to \infty$, $nb_{Q_V}^4 \to \infty$, $_n n b_V^6 < \infty$, $_n nb_{Q_V}^6 < \infty$.

(2.4) $h_V \to 0$, $nh_V^6 \to \infty$, $_n nh_V^8 < \infty$.

(2.5) $_n n^{1/2} b_V m \ 1 < \infty$, $_n n^{1/2} b_{Q_V} m \ 1 < \infty$, $_n n^{1/2} h_V m \ 1 < \infty$.

$t_{ij} \ i=1,\ldots,n \ j=1,\ldots,m$

i $j = 1, \ldots, m \ 1$, $t_{ij} < t_{i,j+1}$ f $\int_{\mathcal{T}} f(t)\,dt = 1$ $_{t \in \mathcal{T}} f(t) > 0$ t_{ij} $t_{ij} = F^{\ 1}(\frac{j \ 1}{m \ 1})$, $F^{\ 1}$ $F(t) = \int_{a_1}^{t} f(s)\,ds$.

t_{ij} i, c_1 c_2 $0 < c_1 < \ _i \ _{t \in \mathcal{T}} f_i(t) < \ _i \ _{t \in \mathcal{T}} f_i(t) < c_2$. f_i N_i i $m \to \infty$ c_1 c_2, $0 < c_1 < \ _i \frac{N_i}{m} < \ _i \frac{N_i}{m} < c_2 < \infty$; $N_i = m$, m

$n = \ t_{ij} \quad t_{i,j\ 1} : j = 2, \ldots, m$,

(3) $n = O(m\ ^1)$, $n, m \to \infty$.

X_{ij} Z_{ij} $t \in \mathcal{T}$,

(4) $E\ X_{ij}^4 < \infty$ $E\ Z_{ij}^4 < \infty$.

H H (1989). $(f \quad g)(h) = \langle f, h\rangle g$ $f, g, h \in H$, H H F $\sigma_2(H)$, $\langle T_1, T_2\rangle_F = (T_1 T_2) = \sum_j \langle T_1 u_j, T_2 u_j\rangle_H$ $T\ _F^2 = \langle T, T\rangle_F$, $T_1, T_2, T \in F$, T_2 T_2, $u_j : j \quad 1$ H. $\mathbf{G}_V$ (6) $\mathbf{G}_V$, G_V (5), G_V (.3), H $\mathcal{I}_i = \ j : \rho_j = \rho_i$ $\mathcal{I}' = \ i : \mathcal{I}_i = 1$, $\mathcal{I}_i$ $\mathcal{I}_i$. $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} \psi_k \quad \psi_k$ $\mathbf{P}_j^V = \sum_{k \in \mathcal{I}_j} \psi_k \quad \psi_k$ H $\psi_k : k \in \mathcal{I}_j$ $\psi_k : k \in \mathcal{I}_j$ j,

$$\delta_j^V = \frac{1}{2} \quad \rho_l \quad \rho_j : l \notin \mathcal{I}_j, \qquad (.1)$$

$\Lambda_{\delta_j^V} = \ z \in \mathcal{C} : z \quad \rho_j = \delta_j^V$, $\mathcal{C}$ $\mathbf{G}_V$ $\mathbf{G}_V$ $\mathbf{R}_V$ $\mathbf{R}_V$, $\mathbf{R}_V(z) = (\mathbf{G}_V \quad zI)^{\ 1}$ $\mathbf{R}_V(z) = (\mathbf{G}_V \quad zI)^{\ 1}$.

$$A_{\delta_j^V} = \left\{ \mathbf{R}_V(z)\ _F : z \in \Lambda_{\delta_j^V} \right\} \qquad (.2)$$

$M = M(n)$ $V(t)$, $V_i(t) = \mu_V(t) + \sum_{m=1}^{M(n)} \zeta_{im}\psi_m(t)$ (13) . $\pi\ _\infty = \ _{t \in \mathcal{T}}\ \pi(t)$ $\pi(\ )$ $\mathcal{T}$. μ_V ψ_j $M = M(n)$ n m $n \to \infty$,

(5) $\tau_n = \sum_{j=1}^M (\delta_j^V A_{\delta_j^V}\ \psi_j\ _\infty)/(\ \bar{n} h_V^2 \quad A_{\delta_j^V}) \to 0$, $M = M(n) \to \infty$;

(6) $\sum_{j=1}^M\ \psi_j\ _\infty = o(\ \bar{n} b_V, \ \bar{m}\ )$ $\sum_{j=1}^M\ \psi_j\ _\infty\ \psi_j'\ _\infty = o(m)$.

(5) (6) ψ_j j $\theta_m = b_S^2 + (\ \bar{m} b_S)^{\ 1}$, V :

(7) $E\ _{t \in \mathcal{T}}\ V(t) \quad V^{(M)}(t)\ ^2 = o(n)$, $V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^M \zeta_k \psi_k(t)$.

(8) 1 i n, $\theta_m \sum_{k=1}^M\ \psi_k\ _\infty^2 = o_p(1)$ $\gamma_n = \sum_{k=1}^M (\delta_k^V A_{\delta_k^V}\ \psi_k\ _\infty)/(\theta_m^{\ 1} \quad A_{\delta_k^V}) \to 0$ $n \to \infty$.

$t = t_{ij}, t_1 = t_{ij_1}$, $t_2 = t_{ij_2}$, i, j, j_1, j_2, $g(x\ t)$ X_{ij} $g_2(x_1, x_2\ t_1, t_2)$ (X_{ij_1}, X_{ij_2}). $f(z\ t)$ $f_2(z_1, z_2\ t_1, t_2)$ Z_{ij} (Z_{ij_1}, Z_{ij_2}). $g(\ t), f(\ t), t \in \mathcal{T}$, $g_2(\ t_1, t_2), f_2(\ t_1, t_2), t_1, t_2 \in \mathcal{T}$, :

(1.1) $(d^2/dt^2) g(x\ t)$ $(d^2/dt^2) f(z\ t)$ $\Re$ $\mathcal{T}$.

(1.2) $(d^2/dt_1^{\ 1} dt_2^{\ 2}) g_2(x_1, x_2\ t_1, t_2)$ $(d^2/dt_1^{\ 1} dt_2^{\ 2}) f_2(z_1, z_2\ t_1, t_2)$ $\Re^2$ $\mathcal{T}^2$, $_1 + \ _2 = 2, 0 \quad _1, \ _2 \quad 2$.

$\kappa_1 : \Re \to \Re$ $\kappa_2 : \Re^2 \to \Re$ $\kappa_1(u)$ $\kappa_2(u, v)$ $\chi_1(t) = \int e^{\ iut} \kappa_1(u)\, du$ $\chi_2(t, s) = \int e^{\ (iut + ivs)} \kappa_2(u, v)\, du\, dv$. :

(2.1) κ_1 $\kappa_1\ ^2 = \int \kappa_1^2(u)\, du < \infty$, $\chi_1(t)$ $\int\ \chi_1(t)\ dt < \infty$.

(2.2) κ_2 $\kappa_2\ ^2 = \int\int \kappa_2^2(u, v)\, du\, dv < \infty$, κ_2 0 u v. $\chi_2(t, s)$ $\int\int\ \chi_2(t, s)\ dt\, ds < \infty$.

APPENDI C: PROOF

P n 1

W S V, $= \ _1 \ _2$. E ^{2} (S V), $\omega_1 \in \ _1$ ω_1, $R_j = R_j(\omega_1)$ i R_{ij} (1) ^{2} j, $E\ (R_j) = 0$ $E\ (R_j^2) < C_1$ C_1. (1) ^{3} ^{2} (1979)

$$E\ \left(\ _{t \in \mathcal{T}}\ S(t) \quad S(t)\ \right)(\omega_1) = O\left(b_S^2 + \frac{1}{\bar{m} b_S}\right)(\omega_1).$$

ω_1, $S^{(v)}(\omega_1)$, $v = 0, 1, 2$, $V(\omega_1)$ (1), ω_1;

$$\ _{\omega_1 \in \ _1}\ E\ \left(\ _{t \in \mathcal{T}}\ S(t) \quad S(t)\ \right)(\omega_1) = O\left(b_S^2 + \frac{1}{\bar{m} b_S}\right),$$

(14).

.1. V t_{ij}, Z_{ij} ( ) μ_V, G_V, σ_W^2, ψ_k, ρ_k, ζ_{ik}, (.2), (.3), (.4), (.5)

$$\alpha_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\bar{n} h_V^2 \quad A_{\delta_k^V}} \qquad \alpha_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\theta_m^{\ 1} \quad A_{\delta_k^V}}, \qquad (.1)$$

$\theta_m = b_S^2 + (\ \bar{m} b_S)^{\ 1}$ δ_k^V $A_{\delta_k^V}$ (.1) (.2).

Lemma C.1. (2.1), (2.3), (3), (5), (1.1), (2.2),

$$\ _{t \in \mathcal{T}}\ \mu_V(t) \quad \mu_V(t)\ = O_p\left(\frac{1}{\bar{n} b_V}\right)$$

$$\ _{s, t \in \mathcal{T}}\ G_V(s, t) \quad G_V(s, t)\ = O_p\left(\frac{1}{\bar{n} h_V^2}\right). \qquad (.2)$$

ρ_k … 1, ψ_k …

$$\sup_{t\in\mathcal{T}} |\hat\psi_k(t) - \psi_k(t)| = O_p(\alpha_{nk}) \quad \text{and} \qquad (.3)$$
$$|\hat\rho_k - \rho_k| = O_p(\alpha_{nk}),$$

… $\alpha_{nk} \to 0$ … $n \to \infty$, k …, α_{nk} … (.1), … $O_p(\,)$ … (.3) … 1 … k … M. … (.2),

$$\sup_{t\in\mathcal{T}} |\hat\sigma^2_W(t) - \sigma^2_W(t)| = O_p\left(\max\left\{ \frac{1}{\bar{n}h_V^2}, \frac{1}{\bar{n}b_{Q_V}} \right\}\right). \qquad (.4)$$

… (1), (7) … (1.1), (2.2),

$$\sup_{1\le k\le M} |\hat\zeta_{ik} - \zeta_{ik}| \xrightarrow{p} 0 \quad \text{and} \qquad (.5)$$
$$\sup_{t\in\mathcal{T}} \left| \sum_{k=1}^{M} \hat\zeta_{ik}\hat\psi_k(t) - \sum_{k=1}^{\infty} \zeta_{ik}\psi_k(t) \right| \xrightarrow{p} 0,$$

… M … $M = M(n) \to \infty$ … $n \to \infty$.

Proof of Lemma C.1. … (.2), (.3), … (.5) …
… 1 … 1 … (2006). … σ_W^2

$$+ \sup_{t\in\mathcal{T}} \left\{ \left| \sum_{k=1}^{M} \zeta_{ik}\psi_k(t) \quad \sum_{k=1}^{\infty} \zeta_{ik}\psi_k(t) \right| \right\}$$

$$Q_{i1}(n) + Q_{i2}(n),$$

[illegible] $Q_{i1}(n) \xrightarrow{p} 0$ [illegible] $Q_{i2}(n) \xrightarrow{p} 0$. [illegible] (.5), [illegible] $Q_{i2}(n) \xrightarrow{p} 0$ [illegible] (1), (7), [illegible] $Q_{i2}(n) = O(\vartheta_{in}^{(2)})$, [illegible] $O(\ )$ [illegible] $1 \le i \le n$. [illegible] $Q_{i1}(n)$,

$$Q_{i1}(n) \le \sup_{t\in\mathcal{T}} \left\{ \sum_{k=1}^{M} \zeta_{ik} \quad \zeta_{ik} \quad \psi_k(t) \right.$$

$$\left. + \sum_{k=1}^{M} \zeta_{ik} \quad \psi_k(t) \quad \psi_k(t) \right\}. \quad (.11)$$

[illegible] (.10), [illegible] (.11) [illegible]

$$C_1\theta_{im} \sum_{k=1}^{M} \psi_k \,{}^2_\infty + \gamma_n \left\{ C_2 + \sum_{j=2}^{m} Z_{ij}\,(t_{ij} \quad t_{i,j \ \ 1}) \right\} \xrightarrow{p} 0.$$

[illegible] (.11) [illegible] $O_p\ \sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E\,\zeta_{ik}\ /(\theta_m^{\ 1} \quad A_{\delta_k^V})$. [illegible] $E\ \sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E\,\zeta_{ik}\ /(\theta_m^{\ 1} \quad A_{\delta_k^V})$ [illegible] $\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V}\ \overline{\rho_k}/(\theta_m^{\ 1} \quad A_{\delta_k^V})$ [illegible] γ_n, [illegible] $\rho_k \to 0$, [illegible] 0 [illegible] $Q_{i1}(n) = O_p(\vartheta_{in}^{(1)})$, [illegible] $O_p(\ )$ [illegible] i. [illegible] (18) [illegible] $\sup_{t\in\mathcal{T}} V_i(t) \quad V_i(t) = O_p(\vartheta_{in}^{(1)} + \vartheta_{in}^{(2)})$, [illegible] $O_p(\ )$ [illegible] $1 \le i \le n$.

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