Journal of Economic Dynamics & Control 154 (2023) 104711

Contents lists available at ScienceDirect

Journal of Economic Dynamics and Control

journal homepage: www.elsevier.com/locate/jedc

Social contagion and the survival of diverse investment styles ☆

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ARTICLE INFO

G40
G11
G12
G23

Contagion
Investment styles
Investor behavior
Investor psychology
Adaptive markets

ABSTRACT

We examine the contagion of investment ideas in a multiperiod setting in which investors are more likely to transmit their ideas to other investors after experiencing higher payoffs in one of two investment styles with different return distributions. We show that heterogeneous investment styles are able to coexist in the long run, implying a greater diversity than predicted by traditional theory. We characterize the survival and popularity of styles in relation to the distribution of security returns. In addition, we demonstrate that psychological effects such as conformist preference can lead to oscillations and bubbles in the choice of style. These results remain robust under a wide class of replication rules and endogenous returns. They offer empirically testable predictions, and provide new insights into the persistence of the wide range of investment strategies used by individual investors, hedge funds, and other professional portfolio managers.

1. Introduction

The Efficient Markets Hypothesis (Samuelson, 1965; Fama, 1970) maintains that market prices fully reflect all publicly available information. It is based upon the premise that there are market participants who will take advantage of any mispricing, and that investors with correct beliefs will grow richer at the expense of agents with incorrect beliefs (Fama, 1965). In consequence, markets will be dominated by agents with accurate beliefs about prices (Alchian, 1950; Friedman, 1953).

However, an accumulation of evidence from psychology, cognitive science, behavioral economics, and finance has documented significant violations of individual rationality and the Efficient Markets Hypothesis. In particular, there is evidence of social contagion of investment behavior in financial markets that is not always explained by rational information processing.[4] To better understand

☆ We thank Alex Chinco, Thorsten Hens, David Sraer, Liyan Yang (editor), an anonymous reviewer, and conference and seminar participants at the 2022 China International Conference in Finance, the 2022 Asian Finance Association Annual Conference, the 2022 Bachelier Finance Society World Congress, the 2018 conference on Evolution and Financial Markets, New York University, and Peking University for very helpful comments. Research support from the National Key R&D Program of China (2022YFA1007900), the National Natural Science Foundation of China (12271013), the Fundamental Research Funds for the Central Universities (Peking University), and the MIT Laboratory for Financial Engineering is gratefully acknowledged.

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[4] Examples of social contagion include evidence from stock markets (Hong et al., 2004; Ivković and Weisbenner, 2007; Brown et al., 2008; Kaustia and Knüpfer, 2012; Ozsoylev et al., 2014; Ammann and Schaub, 2021), mutual funds and hedge funds (Hong et al., 2005; Cohen et al., 2008; Boyson et al., 2010; Pool et al., 2015; Kuchler et al., 2022), and housing markets (Burnside et al., 2016; Bailey et al., 2018); see also the review of Hirshleifer and Teoh (2009) and the discussions of social economics and finance of Shiller (2017) and Hirshleifer (2020).

https://doi.org/10.1016/j.jedc.2023.104711
Received 1 April 2023; Received in revised form 8 June 2023; Accepted 13 July 2023

Available online 25 July 2023

empirically tested by examining shifts in investment style such as value versus growth, momentum versus contrarian, or fundamental versus quantitative as a function of market volatility.

More generally, our model helps to explain and predict the survival of a diverse range of investment styles given their return characteristics. For example, there are numerous categories of hedge funds with widely varying investment styles (Chan et al., 2006). The hedge fund sector is subject to intense selection pressure, and has been called the "Galápagos Islands" of finance (Lo, 2008).[9] Darwin's original 1835 observations in the Galápagos Islands suggested that environmental segmentation was the source of evolutionary diversification. In fact, our framework suggests that diversity can persist even within a single non-partitioned environment, a surprising but important distinction in market evolution.

We check the robustness of the implications of our model by considering several extensions. First, we allow for very general rules of replication that are increasing functions of realized returns, capturing the intuitions that higher returns benefit the spread of an investment philosophy. We find that higher variances of a style promote survival when replication functions are convex such that they represent lottery-like preferences. Second, return distributions are exogenous in our basic model, which we extend by considering market equilibrium with endogenous returns in the spirit of Lux's (1995) classical model. When more investors adopt a philosophy, the demand for the stocks that this investment philosophy calls for buying increases. This demand is cleared in the market with suppliTc(benefit)Tj/F2 1 Tf9.9626 0 030 Tc( )Tj/F1 1 Tf7.9701 0 0 7.97s1 Tf9.9626 0 0 9.9626 164.5811 expense of its hosts. This different focus motivates us to not only generalize the rules of replication between two generations as functions of high payoffs may tend to talk more about their returns with other investors, a phenomenon that Han et al. (2022) refer to as a self-enhancing transmission bias. Regardless of the channel, investors with higher realized returns will produce more offspring with the same philosophy ( ) as themselves in the next period. We therefore make the following simple assumption.[17]

Assumption 3. or is also the number of offspring generated by the investment style or , respectively.

Hence, the number of offspring of individual , is given by:

$$= I + (1 - I) \quad , \quad I \equiv \begin{cases} 1 & \text{with probability} \\ 0 & \text{with probability } 1 - \; . \end{cases} \tag{1}$$

We assume that the trait value is passed on without modification to newly infected individuals. As a result, the population may be viewed as being composed of "types" of individuals indexed by values of that range from 0 to 1.

Equation (1) provides the model with the basis for its insights into the evolution of investor types over many generations, since it signifies the dynamics between periods. In focusing on the evolution of the distribution of types in the population, it differs from the large body of literature that instead focuses on the evolution of the distribution of wealth across investors.[18] The reproducing units in our framework are not investors or traders, but instead instances of investment philosophies. This has two implications.

First, one generation in our evolution model should not be interpreted as the actual lifetime of an investor, but rather the duration over which investment ideas spread. This can be weeks, days, or even minutes, as information spreads over social networks with modern information technology.

Second, our model can be modified to describe the switching of philosophies in a population of long-lived investors, as long as is normalized by the total number of investors in the population. With this interpretation, investors' behaviors may depend on historical information beyond the returns in the t h e

Proposition 1. ... f^* ...

$$f^* = \begin{cases} 1 & \mathbb{E}[\ /\ ] < 1 \\ \ & \mathbb{E}[\ /\ ] \geq 1 \quad \mathbb{E}[\ /\ ] \geq 1 \\ 0 & \mathbb{E}[\ /\ ] < 1, \end{cases} \tag{5}$$

f^* ... fi ...

$$\mathbb{E}\left[\frac{\ -\ }{\ f^*\ + (1 - f^*)\ }\right] = 0 \tag{6}$$

... $\Phi(\ ,\ )$.

The growth-optimal type f^* is a function of the financial environment $\Phi(\ ,\ )$. The role of Φ is critical in our framework, as it completely characterizes the effect of an investor's actions upon the type's reproductive success. The growth-optimal type f^* dominates the population in the long run because it grows exponentially faster than any other type. We will refer to f^* as the evolutionary equilibrium philosophy. It emerges through the forces of natural selection quite differently from the neoclassical economic framework of expected utility optimization.[19] in the evolutionary biology literature (Cooper and Kaplan, 1982; Frank and Slatkin, 1990; Frank, 2011).[20]

Proposition 1 holds for any return distribution $\Phi(\ ,\ )$ that satisfies Assumptions 1–2. However, it is interesting to give $(\ ,\ )$ a factor structure, and study how the contagion of investment ideas across investors affects the equilibrium investment philosophy f^*.

Let be the common component of returns shared by styles and , ϵ and ϵ the style-specific components, and μ and μ the mean returns of styles and .

Assumption 4. The gross returns to the two styles are

$$= \mu + \beta\ + \epsilon$$

$$= \mu + \beta\ + \epsilon,$$

where $\beta > 0$ and $\beta > 0$ are the sensitivity of style returns to the common return component; , ϵ and ϵ are independent and bounded random variables such that and are always positive; and $\mathbb{E}[\ ] = \mathbb{E}[\epsilon\ ] = \mathbb{E}[\epsilon\ ] = 0$.

Assumption 4 allows for a very wide set of possible investment styles. For instance, the two styles could be active versus passive investments, value versus growth stocks, fundamental versus quantitative strategies, domestic versus global investment, large firm versus small firm, long-only versus long-short, single-factor vs. multi-factor, and so forth. Different assumptions about the characteristics of μ, β, ϵ (where $=\ ,$), and imply different cases of interest.

4. Evolutionary survival of investment styles

We next ask the question: how does the evolutionary equilibrium investment philosophy depend on the style return characteristics, including its expected returns, return betas, and return variances? We first identify the conditions for an equilibrium to consist solely of the choice of a single style, and then study the case where the long-run equilibrium population consists of investors who adopt both styles with positive probability. We briefly refer to empirical testing, but this topic is covered more extensively in Section 8 and Appendix E.

By Proposition 1, the expected value of the ratios / and / determines whether the evolutionary equilibrium investment philosophy involves only one style, or a combination of the two. Let $\equiv\ /\ $, so that

$$\mathbb{E}[\ ] = \mathbb{E}\left[\frac{\ }{\ }\right] = \mathbb{E}\left[\frac{\mu + \beta\ + \epsilon}{\mu + \beta\ + \epsilon}\right], \tag{7}$$

$$\mathbb{E}[1/\ ] = \mathbb{E}\left[\frac{\ }{\ }\right] = \mathbb{E}\left[\frac{\mu + \beta\ + \epsilon}{\mu + \beta\ + \epsilon}\right]. \tag{8}$$

[19] In fact, the evolutionary framework does not require a utility function initially, and the utility function itself can be endogenously determined by natural selection, as shown by Zhang et al. (2014).

[20] See Lo et al. (2021) for experimental evidence in the context of financial decision making.

For corner solutions, we focus on the case where style dominates the population ($^* = 1$). The case where style dominates the population ($^* = 0$) is similar. It is obvious from (8) that the following comparative statics on the conditions for $^* = 1$ apply:

Proposition 2 *( ).* $\mathbb{E}[1/\ ] < 1$, ($\mathbb{E}[1/\ ]$)

(i) , μ ,

(ii) , μ ,

It is not surprising that a higher expected return of a style will promote its dominance in the population. To derive results for other return characteristics, we need to better understand $\mathbb{E}[\ ]$ and $\mathbb{E}[1/\ ]$. Applying the Taylor approximation of as a function of , ϵ and ϵ to estimate (7)-(8) we obtain

$$
\begin{aligned}
(,\epsilon ,\epsilon) = \frac{}{} &= \frac{\mu + \beta + \epsilon}{\mu + \beta + \epsilon} \\
&= (0,0,0) + \frac{\partial\ _0}{\partial} + \frac{\partial\ _0}{\partial \epsilon}\epsilon + \frac{\partial\ _0}{\partial \epsilon}\epsilon \\
&\quad + \frac{1}{2}\left(\frac{\partial^2\ _0}{\partial\ ^2}\ ^2 + \frac{\partial^2\ _0}{\partial \epsilon^2}\epsilon^2 + \frac{\partial^2\ _0}{\partial \epsilon^2}\epsilon^2 + 2\frac{\partial^2\ _0}{\partial\ \partial \epsilon}\ \epsilon + 2\frac{\partial^2\ _0}{\partial\ \partial \epsilon}\ \epsilon + 2\frac{\partial^2\ _0}{\partial \epsilon\ \partial \epsilon}\epsilon\ \epsilon \right) + (\ ^2, \epsilon^2, \epsilon^2).
\end{aligned}
$$

After taking the expected value of , the linear terms vanish, since $\mathbb{E}[\ ] = \mathbb{E}[\epsilon\ ] = \mathbb{E}[\epsilon\ ] = 0$. The second-order cross terms also vanish because , ϵ and ϵ are independent. Therefore, $\mathbb{E}[\ ]$ can be approximated by $(0,0,0)$ and the second-order terms $(\ )$, $(\epsilon\ )$ and $(\epsilon\ )$. A similar approximation applies for $\mathbb{E}[1/\ ]$, which is summarized in the following:

Lemma 1. , , ϵ ϵ

$$
\mathbb{E}[\ ] = \mathbb{E}\left[\frac{}{}\right] \approx \frac{\mu}{\mu} + \frac{\beta\ \beta^2}{\mu^3}\left(\frac{\mu}{\beta} - \frac{\mu}{\beta}\right) (\ ) + \frac{\mu}{\mu^3} (\epsilon\ ),
$$

$$
\mathbb{E}[1/\ ] = \mathbb{E}\left[\frac{}{}\right] \approx \frac{\mu}{\mu} + \frac{\beta^2 \beta}{\mu^3}\left(\frac{\mu}{\beta} - \frac{\mu}{\beta}\right) (\ ) + \frac{\mu}{\mu^3} (\epsilon\ ).
$$

We define $\mu\ /\beta$ and $\mu\ /\beta$ as a style's , which plays a critical role in determining the comparative statics for return beta and volatility, as shown in the next two propositions.

The scaled alpha has an interesting analogy to the slope of the security market line in the Capital Asset Pricing Model. In that model, all investor portfolios satisfy the same security market line slope, $(\overline{R} - R_F)/\beta$, where $\overline{R}$ is the investor's mean (net) return, R_F is the risk-free rate of return, and β is the portfolio's sensitivity to the return on the market. In our model, μ and μ are gross returns, and the scaled alpha can be decomposed into

$$
\frac{\mu}{\beta} = \frac{1 + \overline{R}}{\beta} = \frac{\overline{R} - R_F}{\beta} + \frac{1 + R_F}{\beta}.
$$

Therefore, if CAPM holds, the scaled alpha for the two investment styles differs only by $(1 + R_F)/\beta$. In the same market where R_F is a constant, this is determined by a style's beta, so that beta becomes the key determinant of strategy survival. The importance of scaled alpha will become clear after the following results.

Proposition 3 *( ).* $\mathbb{E}[1/\ ] < 1$. , ϵ ϵ , ($\mathbb{E}[1/\ ]$)

(i) , β ,

(ii) , β , *ffi*

$$
\frac{\mu\ /\beta}{\mu\ /\beta} > 2;
$$

(iii) , β , *ffi*

$$
\frac{\mu\ /\beta}{\mu\ /\beta} < 2.
$$

The conditions for style to dominate in the population are not symmetric with respect to β and β . First of all, a higher β will always promote the dominance of style . Intuitively, this is because the log-geometric average growth rate in Equation (4) is nonlinear with respect to returns, and therefore the upside and downside for style 's realized returns do not offset. As a result, the high systematic risk of the competing style promotes the success of style because the risk causes near-extinctions of style in the market selection process.

However, this is not always the case for β. For the same reason as described above, the high systematic risk of style reduces its own success, but this is only true conditionally on style 's scaled alpha being comparable to or smaller than style 's. If the reverse is true, that is, if the mean return on style is sufficiently strong relative to its risk (if style 's scaled alpha is sufficiently higher than style 's), the higher β actually encourages the dominance of style in the population. In other words, style 's high scaled alpha serves as protection from its own downside risk.

We provide intuitions behind the asymmetry between β and β. Although styles and are symmetric in our model, Proposition 3 provides conditions under which - tend to dominate. From their perspective, the two styles are not symmetric because β represents the influence from its beta while β represents the influence from the style's beta. Mathematically, this reflects the nonlinearity in boundary conditions in Equations (7)–(8). In fact, if one considers conditions under which style -investors tend to dominate, all results will be symmetric relative to Proposition 3, by replacing styles with style .

Proposition 4 *( ).* , - $\mathbb{E}[1/\ ] < 1$. - , ϵ ϵ, (, $\mathbb{E}[1/\ ]$)

(i) - fi , (ϵ),

(ii) , ( ), ' '

$$\frac{\mu}{\beta} > \frac{\mu}{\beta}$$

(iii) , ( ), ' '

$$\frac{\mu}{\beta} < \frac{\mu}{\beta}$$

Investment style tends to dominate if its idiosyncratic variance is small, for essentially the same reason discussed earlier for return betas. A high variance tends to work against a style because of the nonlinearity of the long-term growth, as reflected in Equation (4); the upside and downside for style 's realized returns fail to offset.

Again, since we are considering the conditions for style to dominate in this case, the results are not symmetric with respect to the idiosyncratic variances of style and style . It is interesting that style 's idiosyncratic variance does not affect style 's dominance (up to a second-order Taylor approximation).

The directional dependence on the variance of the common component is determined by the scaled alpha. A higher variance of the common component encourages style to be dominant only if its scaled alpha is higher than style 's. Intuitively, a higher ( ) increases the variance of both investment styles, and the overall effect therefore depends on the relative sizes of the betas of both styles. However, the effect of risk also depends on the mean return. A high mean return acts as a buffer that reduces the importance of risk. It is therefore the scaled alpha that matters, not merely beta.

Propositions 2–4 together give a complete picture of the comparative effects on the conditions of $^* = 1$ (that is, always choosing style) for mean returns, return betas, and return variances. Parallel results can also be derived for $^* = 0$ (always choosing style) using approximations for $\mathbb{E}[\ ]$ in Lemma 1 instead. In the next section, we discuss mixed survival of investment styles.

. .

In general, if the evolutionary equilibrium philosophy involves both investment styles, * is given by (6). With Assumption 4, the first-order condition becomes:

$$\mathbb{E}\left[\frac{(\mu\ -\mu\ )+(\beta\ -\beta\ )\ +\epsilon\ -\epsilon}{[\ \mu\ +(1-\ )\mu\ ]+[\ \beta\ +(1-\ )\beta\ ]\ +[\ \epsilon\ +(1-\ )\epsilon\ ]}\right]=0. \tag{9}$$

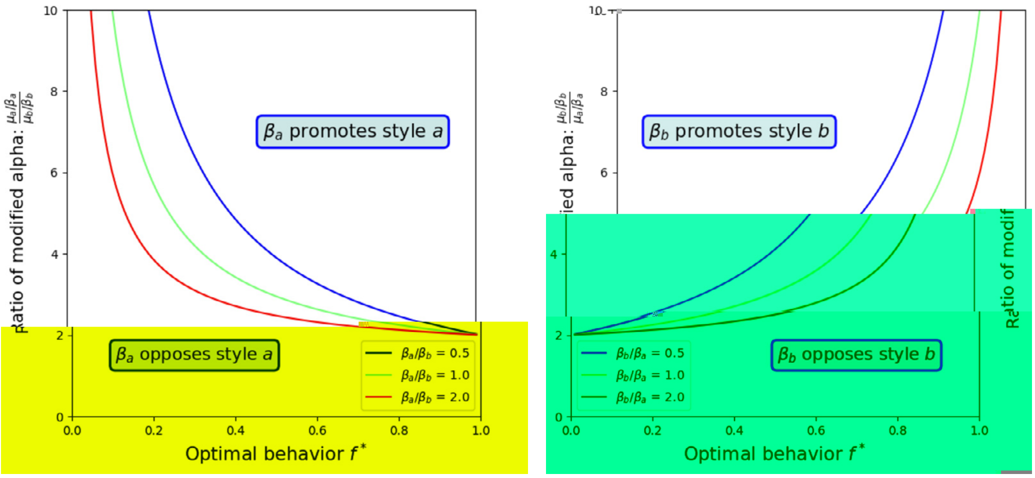
Taking derivatives of Equation (9) to μ and μ, we immediately have the following comparative statics for the philosophy *.

Proposition 5 *( ).* , *

(i) , μ ,

(ii) , μ ,

Not surprisingly, Proposition 5 is similar to Proposition 2; they both assert that a higher expected return encourages investment in that style. To empirically test Propositions 2 and 5, one can estimate historical mean returns of value versus growth stocks, and see if a change in their realized returns over time corresponds to change in the frequencies of value versus growth investors. These can be estimated, e.g., from mutual fund holdings or social media data. In the context of hedge funds, one can look at the average



(a) Comparative statics for β_a (b) Comparative statics for β_b

Fig. 1. Comparative Statics for Return Beta: β (1a) and β (1b). In the case of β (1a), the vertical axis represents the ratio of the scaled alpha, $\frac{\mu/\beta}{\mu/\beta}$, and the horizontal axis represents the evolutionary equilibrium philosophy, *. Three lines of different colors represent the boundaries between promoting and demoting style for three different ratios of beta, β/β. The upper region represents when β promotes style , while the lower region represents when β opposes style . The case of β (1b) is symmetrical.

return of different investment styles, such as fundamental versus quantitative in a certain period, and correlate that with attrition rates in different hedge fund categories. Section 8 and Appendix E discuss directions for empirical tests in more detail.

To derive further comparative statics, we again use a Taylor expansion to approximate the first-order condition of Equation (9).

Lemma 2. , ϵ ϵ, fi ( )

$$0=(\mu-\mu)[\ \mu+(1-\ )\mu]^2+\beta\beta[\ \beta+(1-\ )\beta]\left(\frac{\mu}{\beta}-\frac{\mu}{\beta}\right) \quad ( )$$

$$+(1-\ )\mu\ (\epsilon)-\ \mu\ (\epsilon).$$

When $\mathbb{E}[\ /\ ]\geq 1$ and $\mathbb{E}[\ /\ ]\geq 1$, the evolutionary equilibrium philosophy involves mixed investment styles, and * is given by Lemma 2, up to a second-order Taylor approximation.

Proposition 6 ( ). , ϵ ϵ, *

(i) β $\frac{\mu/\beta}{\mu/\beta}>2+\frac{1-\ ^*}{\ ^*}\left(\frac{\beta}{\beta}\right)$

(ii) β $\frac{\mu/\beta}{\mu/\beta}<2+\frac{1-\ ^*}{\ ^*}\left(\frac{\beta}{\beta}\right)$[21]

(iii) β $\frac{\mu/\beta}{\mu/\beta}>2+\frac{\ ^*}{1-\ ^*}\left(\frac{\beta}{\beta}\right)$

(iv) β $\frac{\mu/\beta}{\mu/\beta}<2+\frac{\ ^*}{1-\ ^*}\left(\frac{\beta}{\beta}\right)$.[22]

The relationship between the evolutionary equilibrium philosophy * and return beta is determined by three components: the ratio of scaled alphas, the ratio of betas, and the philosophy *. Fig. 1 shows the regions in which the return beta promotes or opposes the investment style, as a function of the ratio of the scaled alpha and the philosophy *.

Proposition 6 generalizes Proposition 3 from the case of a single dominant style to the case of mixed styles. To see this, suppose style is dominant and $^*=1$. The condition in the fourth item of Proposition 6 is always true, and therefore the dominance tends to occur when β increases, which corresponds to the first item of Proposition 3, and $^*=1$ in Fig. 1b. Similarly, the condition in the first two items of Proposition 6 reduces to the second and third item of Proposition 3 trivially, and this corresponds to $^*=1$ in Fig. 1a. Once again, the scaled alphas μ/β and μ/β play a critical role in determining the direction of beta's impact on the philosophy *. Instead of threshold 2 in Proposition 3, the threshold here is adjusted by a positive amount, the adjustment depending on * and β/β, as shown in Fig. 1.

[21] This is always true when $^*\leq\frac{\mu}{\mu+\mu}$, since the right-hand side reduces to $2+\frac{\mu/\beta}{\mu/\beta}$.

[22] This is always true when $^*\geq\frac{\mu}{\mu+\mu}$, since the right-hand side reduces to $2+\frac{\mu/\beta}{\mu/\beta}$.

Factor sensitivity β always opposes style when, $^* \leq \frac{\mu}{\mu + \mu}$. Intuitively, this means that when the equilibrium frequency of style -investors is small relative to the proportion of style 's expected return $\frac{\mu}{\mu + \mu}$, it promotes the survival of a philosophy to decrease the weight of style as style 's beta increases. On the other hand, when, $^* \geq \frac{\mu}{\mu + \mu}$, it promotes the survival of a philosophy to decrease the weight of style as style 's beta increases, symmetric to the case for β .

When two investment styles have comparable scaled alphas ($\frac{\mu / \beta}{\mu / \beta} \approx 1$), β opposes style and β opposes style . In other words, a lower beta investment style is preferred if its scaled alpha is comparable to other styles in the market. In the context of hedge funds, a testable implication is that a low beta strategy should attract more investors after controlling for factors such as expected return and volatility, especially when the scaled alpha is comparable with alternative investment styles.

This result is derived using exogenous returns (Assumption 4). However, if investors are attracted to the low-beta style, they may drive up its price and drive down its expected return, which tends to have a negative feedback effect on the survival of the low-beta style. Nevertheless, we show that these results hold in a market equilibrium setting with endogenous returns in Section 6.

In contrast, if one investment style has a much higher scaled alpha than the other style (corresponding to the upper regions in Fig. 1), a higher beta actually promotes the popularity of that style. This is because the scaled alpha is so large that it gives substantial downside protection against any increase in variance brought by a higher beta. More variance becomes good for survival in this case. For alternative investments such as hedge funds, private equity and venture capital, the expected return can be very high and the beta can be very low. Therefore, the scaled alpha for these investments can be much higher than that for traditional investment styles. Our model predicts that high-beta styles are favored in this case. In the context of the stock market, this implies that investment styles in high beta stocks will gain popularity if their scaled alphas are sufficiently high, leading to a decrease in returns. In contrast, investment styles in low beta stocks lose popularity, leading to higher returns. This outcome is consistent with the empirical anomaly that low beta stocks earn high expected returns, as contrasted with the traditional risk premium theory that they should earn low expected returns. Our result can therefore justify the use of a common defensive (low-risk) "smart beta" strategy (Frazzini and Pedersen, 2014).

Proposition 7 *( ).* , - , ϵ ϵ , *

(i) - fi , (ϵ),

(ii) - fi , (ϵ),

(iii) , ( ), ' ' $\frac{\mu}{\beta} > \frac{\mu}{\beta}$

(iv) , ( ), ' ' $\frac{\mu}{\beta} < \frac{\mu}{\beta}$.

There will be more style -investors if style 's idiosyncratic variance is small, and if style 's idiosyncratic variance is large. This also generalizes Proposition 4 from the case of a single dominant style to the case of mixed styles. Intuitively, a higher style-specific variance discourages investment in that style,[23] because of the nonlinearity of the long-term growth as reflected in Equation (4). In other words, the possibility of near wipe-outs of an investment style is disproportionately important, opposing the survival of more volatile investment styles.

The directional dependence of the equilibrium philosophy on the variance of the common component is again determined by the scaled alpha. A higher variance of the common component encourages investment in the style with a higher scaled alpha. The reason is similar to that in our previous discussions. A higher ( ) increases the variance of both investment styles, and the overall effect therefore depends on the relative sizes of the betas of both styles. However, the effect of risk also depends on the mean return. A high mean return acts as a buffer that reduces the importance of risk. It is therefore the scaled alpha that matters, not only a comparison of betas.

Proposition 4 and 7 offer interesting new possibilities for the empirical consequences of return variance. In the context of hedge funds, one can test whether high idiosyncratic variance in returns opposes the survival of that investment style, or even specific fund managers with allocations in that style. Industry practitioners often use the Sharpe ratio to select fund managers. If hedge funds truly deliver returns with low correlation to the broader markets, a high Sharpe ratio would directly correspond to low idiosyncratic return variance, consistent with the implications of our model.

Moreover, the effect of the variance of the common component depends on each strategy's scaled alpha. The variance of the common component of two investment styles in general corresponds to the volatility of broader factors such as the market portfolio. This implies that during volatile times, investors with higher scaled alpha tend to flourish. This is directly testable in both individual investment strategies and hedge funds. For example, one can compare the frequency of investors in value versus growth strategies, momentum versus defensive, and so on, during periods of high and low market volatility, and test whether high market volatility promotes survival of those types that invest heavily in styles with high scaled alpha. With hedge fund data, one can study the

[23] In Han et al. (2022) the opposite is true: variance promotes survival. In Han et al. (2022), this effect is driven by a selection bias whereby high returns are more likely to be reported, which is intensified by high variances. The model here allows for a more general distribution in the number of offspring, which results in a distinct intertemporal dynamic effect: a long-run "evolutionary hedging" benefit to avoid very low reproduction outcomes.

attrition rates of different investment styles through different market cycles, testing the similar hypothesis that high market volatility promotes hedge fund categories with high scaled alpha.

We emphasize that our comparative statics results with respect to beta and volatility hold true up to second-order Taylor approximations.[24] These approximations are for and $1/$ in (7)–(8), and ultimately their expected values determine the survival philosophy. As a result, second-order Taylor approximations allow us to derive analytical insights up to the second moment of returns, such as volatility, but they do not account for higher-order moments, such as the skewness of returns, which are left for future studies.

We apply these results to study an example of special return properties in Appendix A.

5. General replication rules

Our basic model has assumed that the replication rule, i.e., the mapping from the returns to the number of offspring, is an identity function (see Assumption 3). Here we consider a general class of replication rules and assess the robustness of the results of our derivations so far.

We first generalize Assumption 3 to allow for a much more general class of replication rules.

Assumption 5. The number of offspring generated by the investment style or is given by $\psi(\ )$ or $\psi(\ )$, where $\psi(\cdot)$ is a that is twice differentiable, non-negative, and non-decreasing:

$$\psi \geq 0 \quad \text{and} \quad \psi' \geq 0.$$

Assumption 6. The replication function is concave: $\psi'' \leq 0$.

Assumption 5 reflects a few natural conditions for any reasonable evolutionary process. $\psi \geq 0$ guarantees that the number of offspring is non-negative. $\psi' \geq 0$ guarantees that higher returns are preferred and therefore do not lead to fewer followers. In Assumption 6, $\psi'' \leq 0$ corresponds to a diminishing marginal effect of return-biased transmission, that is, an increase in returns from 1% to 2% will be more influential than that from 10% to 11%. However, Assumption 6 may not be true for all markets. For example, lottery markets have low expected returns, yet they persistently attract investors. Lottery-like preferences imply that extreme returns attract an overwhelming amount of attention and investments, which is reflected by a convex replication function with $\psi'' > 0$.[25]

By following the same derivations as in Equations (2)–(4), it is easy to show that the average log population for philosophy satisfies:

$$\frac{1}{\ } \log \longrightarrow \mathbb{E}[\log (\ \psi(\ ) + (1 - \ )\psi(\ ))] \equiv \alpha_{\psi}(\ ) \tag{10}$$

as increases without bound. We add the subscript "$_{\psi}$" to the population growth rate $\alpha_{\psi}(\ )$, which emphasizes the fact that ψ determines the growth rate, and therefore, the optimal investment philosophy. The optimal that maximizes (10) is given by:

Proposition 8. , , , $^*_{\psi}$ (0)

$$^*_{\psi} = \begin{cases} 1 & \mathbb{E}\left[\psi(\ )/\psi(\ )\right] < 1 \\ \ & (\ ) \quad \mathbb{E}\left[\psi(\ )/\psi(\ )\right] \geq 1 \quad \mathbb{E}\left[\psi(\ )/\psi(\ )\right] \geq 1 \\ 0 & \mathbb{E}\left[\psi(\ )/\psi(\ )\right] < 1, \end{cases} \tag{11}$$

$^*_{\psi}$ fi ( )

$$\mathbb{E}\left[\frac{\psi(\ ) - \psi(\ )}{\ ^*_{\psi}\psi(\ ) + (1 - \ ^*_{\psi})\psi(\ )}\right] = 0 \tag{12}$$

( )-( ) $\Phi(\ , \ )$.

We can derive a parallel set of comparative statics for $^*_{\psi}$ with respect to return characteristics. In general, the results in Propositions 2–7 are robust to general replication functions ψ, although in certain cases, explicit characterizations of boundary conditions are no longer possible in terms of simple expressions of μ and β. We summarize the key conclusions here, and leave the mathematical details to the proofs in Appendix F.

---

[24] Results with respect to mean returns do not rely on Taylor approximations.

[25] Or at least convex when returns and are high.

We first provide the results for comparative statics when ψ^* is either 0 or 1.

Proposition 9 (...)

(i) (a) ... μ ...
(b) ... μ ...

(ii) (a) β ...
(b) β ... ψ- ...

$$\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} > C_1;$$

(c) β ... ψ- ...

$$\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} < C_1;$$

(iii) (a) ... (ϵ), ...
(b) ... $()$, ... ψ- ...

$$\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} > C_2;$$

(c) ... $()$, ... ψ- ...

$$\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} < C_2.$$

... ϵ ... ϵ. C_1 ... C_2 ...

In Proposition 9, case (i) generalizes Proposition 2, case (ii) generalizes Proposition 3, and case (iii) generalizes Proposition 4.

Next, we relax Assumption 6 to allow for lottery-like preferences. In this case, higher style-specific variances actually promote survival.[26]

Proposition 10 (...) ... $\psi'' > 0$, ...

(i) ... μ ...
(ii) ... (ϵ), ...

... ϵ ... ϵ.

The next result provides the comparative statics when the evolutionary equilibrium philosophy includes both investment styles.

Proposition 11 (...) ... ψ^* ...

(i) (a) ... μ ...
(b) ... μ ...

(ii) (a) β ... $\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} > C_3$
(b) β ... $\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} < C_3$
(c) β ... $\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} > C_3'$
(d) β ... $\frac{\psi(\mu)/\beta}{\psi(\mu)/\beta} < C_3'$.

[26] We thank an anonymous reviewer for suggesting this idea.

(iii) (a) fi , $(\epsilon\ )$,
(b) fi , $(\epsilon\ )$,
(c) , ( ), ' ψ- ffi
' ψ-

$$\frac{\psi(\mu\ )/\beta}{\psi(\mu\ )/\beta} > C_4;$$

(d) , ( ), ' ψ- ffi
' ψ-

$$\frac{\psi(\mu\ )/\beta}{\psi(\mu\ )/\beta} < C_4.$$

( ) ( ) - , ϵ ϵ. C_3, C_3', C_4

In Proposition 11, case (i) generalizes Proposition 5, case (ii) generalizes Proposition 6, and case (iii) generalizes Proposition 7. Like the case of singly dominant rules, when the replication function reflects lottery-like preferences, higher style-specific variances promote survival.

Proposition 12 (-). , , , , - - ffi $\psi'' > C_5$ $C_5 > 0$, , *

(i) , μ ,
(ii) , μ ,
(iii) fi , $(\epsilon\ )$,
(iv) fi , $(\epsilon\ )$,

( ) ( ) - , ϵ ϵ.

Overall, Propositions 9–12 show that the equilibrium philosophy $^*_\psi$ has the same set of dependencies on return characteristics as those for *, with only a different notion of ψ-scaled alpha and a different set of constants specifying its boundary conditions. These results confirm that our key conclusions in Section 4 are robust to a very general class of replication rules.

In addition, by considering general replication rules that are convex, our model provides an explanation for the persistence of lottery markets and lottery-like stocks despite their excess risk, and therefore lower risk-adjusted expected returns. This effect is also consistent with Han et al.'s (2022) model of social transmission bias.

6. Diversity in market equilibrium

So far, we have viewed the returns on investment style as exogenous (Assumption 4). However, in market equilibrium, stock returns reflect shifts in supply and demand as the frequencies of different investment styles shift. In this section, we extend the model to reflect the fact that the imbalance between supply and demand for the securities traded by styles and affects their expected returns. In particular, we build an equilibrium model with endogenous returns and study its implications for the equilibrium investment philosophy.

. .

We start by making a distinction between the fundamental value and the actual price of style and . We interpret and defined in Assumption 4 as gross returns to the processes of style and . We use $P_{,}$ and $P_{,}$ to denote the corresponding fundamental values, which are given by:

$$\begin{aligned} P_{,} &= \cdot P_{,-1}, \\ P_{,} &= \cdot P_{,-1}. \end{aligned} \tag{13}$$

On the other hand, the , $P_{,}$ and $P_{,}$, may deviate from fundamental values due to forces of supply and demand in the market. They are related to the actual returns, R and R, by:

$$\begin{aligned} P_{,} &= R \cdot P_{,-1}, \\ P_{,} &= R \cdot P_{,-1}. \end{aligned} \tag{14}$$

The distinction between fundamental

Solving Equation (19) for market clearing conditions, we have the following result for equilibrium prices and returns.

Proposition 13.

$$P_{,} = P_{,} \left(\frac{\lambda}{F} \right)^{\frac{1}{}}, \qquad P_{,} = P_{,} \left(\frac{(1-\lambda)}{F} \right)^{\frac{1}{}}, \tag{20}$$

$$R_{,} = {}_{,} \left(\frac{\lambda}{\lambda_{-1}} \right)^{\frac{1}{}}, \qquad R_{,} = {}_{,} \left(\frac{1-\lambda}{1-\lambda_{-1}} \right)^{\frac{1}{}}. \tag{21}$$

There are several interesting observations that can be made from Proposition 13. First, the aggregate demand (λ) determines the equilibrium prices and their deviations from the fundamental value, while it is the in aggregate demand between two periods (λ / λ_{-1}) that determines the equilibrium returns. For example, as style generates higher returns, investors with higher will generate more offspring in the next period, driving the aggregate demand in style higher. As a result, we expect the cost of purchasing style securities to increase, which reduces the return for buying and holding style .

Second, the equilibrium prices are affected by the fraction of speculators versus fundamentalists in the market ($_F$/). Because our model does not focus on how this fraction changes over time, the price dynamics are mainly driven by the relative demand (λ).

Third, the exponent 1/ describes the shape of a power-law market impact from trading, which is the reciprocal of the sensitivity to price deviations by the fundamentalists. Higher sensitivities lead to a milder price impact, and lower sensitivities lead to a stronger price impact. This is closely related to Kyle's (1985) market microstructure model in which liquidity is measured by an estimate of the log-volume required to move the price by one dollar.[31]

Finally, if we consider price deviations from the fundamental value:

$$\frac{P_{,}}{P_{,}} = \left(\frac{\lambda}{F} \right)^{\frac{1}{}}, \qquad \frac{P_{,}}{P_{,}} = \left(\frac{(1-\lambda)}{F} \right)^{\frac{1}{}}, \tag{22}$$

our model implies that higher demand in a style (λ) leads to a higher degree of price deviation, what might be considered a bubble, and a higher level of supply sensitivity ( ) makes it more difficult to substantially deviate from the fundamental values, in other words, less likely to form bubbles.

Given the endogenous returns in Proposition 13, we denote an equilibrium philosophy by , with superscript indicating endogenous returns.

Proposition 14. ( ) ( ), $^{*}_{\psi}$.

Proposition 14 shows that though asset prices are affected in the long run by the relative demand in style to style , the equilibrium philosophy remains the same. In other words, our results in Propositions 2–11 remain robust in a model of market equilibrium. This is not surprising given our remarks after Proposition 13. Indeed, equilibrium prices are affected by the aggregate demand in the long run. However, the equilibrium returns of Equation (21) are determined by two terms—the returns on the fundamental value, and an adjustment term that depends on the in demand between two periods. In equilibrium, the second term vanishes to a constant one.

A large literature on market selection has documented that the survival of traders differs markedly from their price influence in the market (Kogan et al., 2006, 2017; Cvitanić and Malamud, 2011; Easley and Yang, 2015). Propositions 13–14 allow us to study whether different surviving philosophies in equilibrium necessarily imply different prices.

[31] See also Bertsimas and Lo (1998), Lillo et al. (2003), and Almgren et al. (2005) for more detailed explorations of the power law of price impact in equity markets.

[illegible]

We consider two scenarios with two different equilibrium philosophies, $_1$ and $_2$, which imply $\lambda_{,1}$ and $\lambda_{,2}$, two equilibrium aggregate demands for style . We follow Easley and Yang (2015) to consider the ratio of equilibrium prices normalized by fundamental values in these two scenarios, $\frac{P_{,,1}/P_{,,1}}{P_{,,2}/P_{,,2}}$ and $\frac{P_{,,1}/P_{,,1}}{P_{,,2}/P_{,,2}}$, where the subscripts 1 and 2 denote the two scenarios. Equation (20) implies that:

$$\begin{aligned} \frac{P_{,,1}/P_{,,1}}{P_{,,2}/P_{,,2}} &= \left(\frac{\lambda_{,1}}{\lambda_{,2}}\right)^{\frac{1}{}} = \left(\frac{_1}{_2}\right)^{\frac{1}{}}, \\ \frac{P_{,,1}/P_{,,1}}{P_{,,2}/P_{,,2}} &= \left(\frac{1-\lambda_{,1}}{1-\lambda_{,2}}\right)^{\frac{1}{}} = \left(\frac{1-_1}{1-_2}\right)^{\frac{1}{}}, \end{aligned} \tag{23}$$

where the right-hand side shows the difference between the two surviving philosophies, while the left-hand side shows the difference between the equilibrium prices.

This relationship shows that, although different philosophies may survive under different style return distributions, their influences on equilibrium prices are milder due to the concavity of the function in Equation (23) when > 1. In our model, represents the elasticity of supply with respect to price deviations from the fundamental value, and more competitive markets imply higher values of . Table A.1 in Appendix B demonstrates this relationship for several different values of . For example, when $= 5$, a two-fold difference in equilibrium philosophies implies a price difference of only 15% at equilibrium.

This phenomenon is similar to that found in Easley and Yang (2015), who find that although market selection in terms of wealth share may be slow for different preferences—in their case, loss aversion versus arbitrageurs—the price impact from investors with loss aversion may be much smaller. We do not model preferences in our framework. Instead, preferences are implicitly reflected by how investors choose between the two investment styles, i.e., the philosophy . Nonetheless, our model highlights a similar phenomenon that market selection in terms of the surviving philosophy and its price impact can be quite varied, especially in competitive markets where the elasticity of supply with respect to price deviations is high.

Appendix B provides two simulated examples to further demonstrate the effect of market equilibrium. In certain cases, market equilibrium in fact speeds up the rate of convergence.

7. Psychological bias and investment philosophy

We have assumed so far that investors are only influenced by the observed payoffs. In reality, investors may also be persuaded to adopt an investment philosophy based upon whether someone else has adopted it. In this section, we discuss two such psychological effects.[32]

[illegible]

Investors may have conformist preferences (Klick and Parisi, 2008), perhaps through the mechanism of viewing other investors as being better informed, and therefore will be influenced by the choices of others. We generalize the population dynamics between two generations in Equation (1) to capture this effect:

$$_{,} = \left[I_{,} \quad + (1 - I_{,}) \quad \right] \exp\left[\tau(\quad - \lambda_{-1})^2\right], \tag{24}$$

where λ_{-1} is the average philosophy in the population in the previous generation -1, and $\tau <= 0$ is the intensity of conformity pressure. When $\tau < 0$, the further is away from the average philosophy λ_{-1}, the more intense is the conformity pressure.

The magnitude of the conformity pressure τ acts roughly as a multiplicative factor in the fitness, or an additive factor in the population growth rate (see Appendix D.1). Suppose a long time has passed, and the evolutionary equilibrium philosophy * that maximizes $\alpha(\ )$ without conformity pressure has dominated the population. The investment philosophy * is evolutionarily stable because any other philosophy grows even more slowly than * with a negative conformity pressure term. However, if * is not initially popular, it may never grow. We verify this implication in the simulation below.

[illegible] We show through a simulated experiment that conformist preference acts as an inertial term that slows down convergence, and in some extreme cases, is even able to change the survival philosophy.

We consider a log-linear specification for the fundamental value process in simulation, which is slightly different from the linear specification in Assumption 4.[33] The fundamental values of the two styles are given by:

---

[32] Psychological factors in which investors' choices depend principally on the behavior of others have been considered in the literature (Lux, 1995; Pedersen, 2022). The key mechanism is similar to Kirman's (1991, 1993) formalization of recruitment in ant populations and Topol's (1991) theory of mimetic contagion.

[33] A linear specification allows us to derive simple closed-form results that highlight the central economic implications of our theory. However, a log-linear specification is convenient in practice because it models and as lognormal distributions, and therefore guarantees that the prices (cumulative returns) do not go negative. The same strategy is also used by Hong et al. (2007).

$$= \exp\left(\mu + \beta + \epsilon - 1\right),$$
$$= \exp\left(\mu + \beta + \epsilon - 1\right), \tag{25}$$

where

$$\mu = \mu = 1, \quad \beta = 2, \quad \beta = 0.1,$$
$$\sim N(0, 0.1^2), \quad \epsilon \sim N(0, 0.3^2), \quad \epsilon \sim N(0, 0.1^2), \tag{26}$$

and N denotes the normal distribution. We set $= 1$, $= 2$, and $_F = 1$ without loss of generality. We simulate the evolution of 11 philosophies in $\{0, 0.1, \cdots, 1\}$. Without any conformity pressure, the equilibrium philosophy is $= 0.5$ for endogenous returns.

Fig. 2 shows the evolution of all philosophies over 20,000 generations. The initial population is composed of 90% $= 0$, and 1% of each $\in \{0.1, 0.2, \cdots, 1\}$. Figs. 2a–2b represent the case of no conformity pressure, showing that $= 0.5$ quickly dominates the population. The price-to-fundamental ratio stays fairly close to one after an initial period of fluctuations.

In comparison, Figures (2c)–2f use different levels of conformity pressure. In the process of convergence to $= 0.5$, other philosophies are popular for extended periods of time. This process may appear as cycles of different popular investment philosophies. Within each period, a certain philosophy is so prevalent in the population that the price-to-fundamental ratios are materially affected, resulting in overpricing for style and underpricing for style . In fact, the popular philosophy in one period could potentially create a long streak of high returns as more investors adopt it, but as the popular philosophy changes, investors holding the previously popular philosophy will quickly be wiped out.

In this example, the initial average philosophy in the population is close to 0, and therefore, philosophies with low will grow more quickly due to the conformity effect. Over time, as the average philosophy λ grows larger, other philosophies start to grow in response. The conformity pressure enhances the survival of the popular philosophy at the time, and inhibits the growth of other philosophies.

In our simulation, the ultimately dominant philosophy has the chance to grow because it begins with a large enough population such that it is never wiped out completely. In reality, philosophies like $= 0.5$ might be eliminated quickly due to conformity pressure. From the evolutionary perspective, mutation would act as insurance for all philosophies to have a chance to grow (see Appendix D.2).

The degree of conformity pressure is likely to be difficult and noisy to measure, but in principle, it can be inferred from textual analysis of social media, or proxies such as the level of adoption of financial innovation (a low amount of innovation might suggest a high degree of conformist preference). Empirical tests for conformist preference could be performed by examining groups with different degrees of conformity pressure, and correlating them with the degree of market efficiency or the speed of convergence after large market shocks.

Opposite in effect to conformist preference is attention to novelty. In attention to novelty, investors are more likely to pay attention to an investment philosophy if it is substantially different from the most popular ones. We modify the population dynamics between two generations in Equation (24) in the following way:

$$= \left[I + (1 - I) \right] \cdot \exp\left[\rho (1 - _{-1}) \right], \tag{27}$$

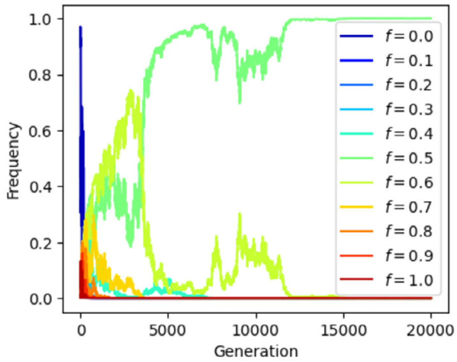
where $_{-1}$ is the population frequency of type- investors in generation -1, defined in (15). Here, $\rho >= 0$ represents the degree of attention to novelty. A higher $_{-1}$ leads to a greater fitness boost due to the attention to novelty.

We next show that attention to novelty can both add diversity and induce bubbles in market evolution. The existence of bubbles, the mechanism through which they form, and the predictability of their formation and collapse have been an active area of research in recent years (Shiller, 2000; Fama, 2014; Greenwood et al., 2019). Our simulation below provides a potential mechanism for the formation of bubbles within our model.

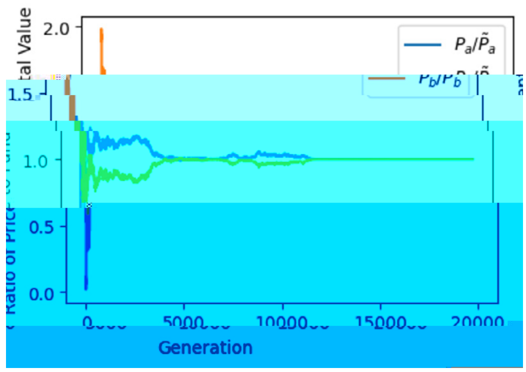
We use the same simulation specifications as in Equation (25) with 11 philosophies in $\{0, 0.1, \cdots, 1\}$. Fig. 3 shows the simulation paths for different degrees of attention to novelty. Figures (3a)–(3b) show the case with no attention to novelty, and $= 0.5$ eventually dominates the population. As the degree of attention to novelty increases to 0.1 in Figure (3c), $= 0.5$ no longer dominates the population. In the long run, there does not exist a single dominant philosophy, because other philosophies are novel compared to the most popular current philosophy and receive a disproportionate conversion in evolution.

In addition, Fig. 3d shows the price-to-fundamental ratio when attention to novelty is set to 0.1. The two investment styles experience repeated episodes of overpricing and underpricing. These patterns of investor composition and asset price dynamics are similar to the bubbles and crashes generated from models of herding (e.g. Lux (1995); Chinco (2023)), as well as return cycles and volatilities generated from learning in markets with multivariate models (e.g. Hong et al. (2007)). Our results provide an alternative channel—attention to novelty—through which such phenomenon can occur.

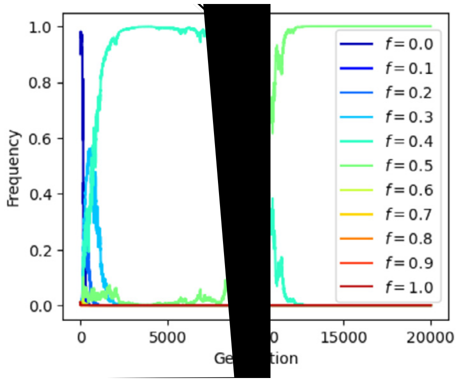
Finally, we consider a variation of the mechanism specified in Equation (27), by allowing the definition of novelty to include memory. In particular, we replace the term $_{-1}$ in Equation (27) by:



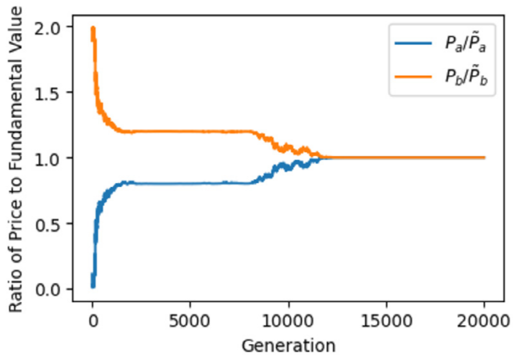
(a) Philosophy Evolution ($\tau = 0$)



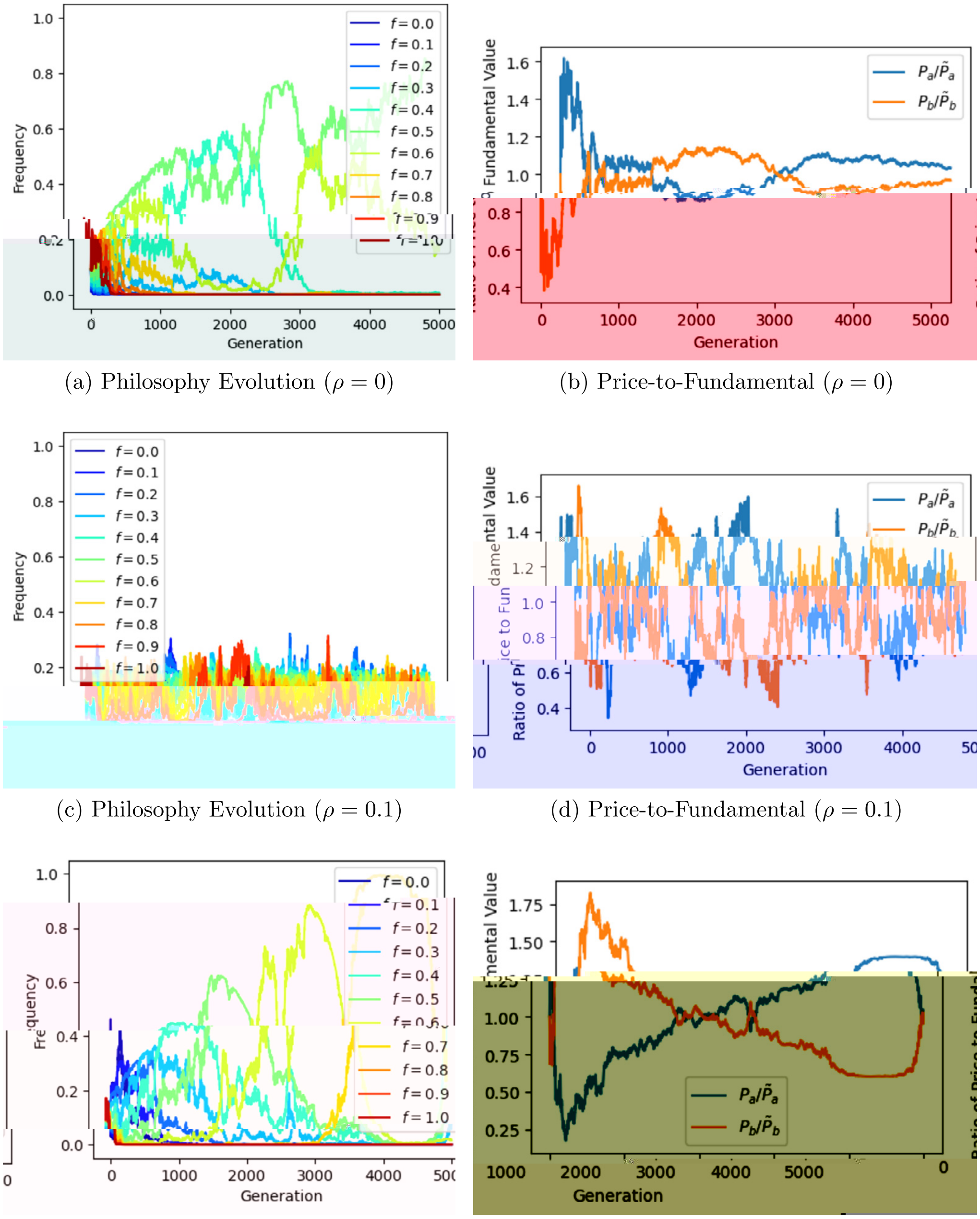
(b) Price-to-Fundamental ($\tau = 0$)



(c) Philosophy Evolution ($\tau = -0.1$)



(d) Price-to-Fundamental ($\tau = -0.1$)



(a) Philosophy Evolution ($\rho = 0$) (b) Price-to-Fundamental ($\rho = 0$)

(c) Philosophy Evolution ($\rho = 0.1$) (d) Price-to-Fundamental ($\rho = 0.1$)

(e) Philosophy Evolution ($\rho = 0.1$, decay=0.9999) (f) Price-to-Fundamental ($\rho = 0.1$, decay=0.9999)

Fig. 3. Evolution of philosophies $\in \{0, 0.1, \cdots, 1\}$ over 5000 generations with the environment (style payoffs) specified in (25)–(26). (3a)-(3b) represent no attention to novelty. (3c)-(3f) represents different degrees of attention to novelty.

8. Summary of empirical implications

We summarize here the key empirical implications of our model in a series of predictions justified by specific aspects of our model. The survival of an investment style or a fund is jointly determined by several elements, including its expected return, beta, and volatility. In particular, the scaled alpha—defined as the expected gross return of a style divided by its beta—plays a critical role.

Prediction 1. A fund with higher expected return tends to attract more investors after controlling for other factors such as beta and volatility.

See Propositions 2 and 5.

Prediction 2. A fund with lower beta tends to attract more investors when its scaled alpha is comparable with alternative funds, and a fund with higher beta tends to attract more investors when its scaled alpha is much higher than alternative funds, both after controlling for other factors such as expected return and volatility.

See Propositions 3 and 6.

Prediction 3. The "beta puzzle"[34] (i.e., that stocks with high beta earn low expected return) tends to occur when market volatility is low.

According to Propositions 4 and 7, stocks with high beta and low expected return have low scaled alphas, which gains popularity when the common variance, ( ), decreases. This drives down the returns for stocks with high beta relative to stocks with low beta.

Prediction 4. A fund with higher idiosyncratic volatility tends to lose investors, and a fund with lower idiosyncratic volatility tends to attract investors, both after controlling for other factors such as expected return, beta, and market volatility.

See Propositions 4 and 7.

Prediction 5. In volatile markets, investors tend to allocate to stocks and funds with higher scaled alphas. A high scaled alpha can therefore be understood as a defensive characteristic of a fund.

See Propositions 4 and 7.

Prediction 6. The "idiosyncratic volatility puzzle"[35] (i.e., that stocks with high idiosyncratic volatility earn low expected return) tends to occur for stocks with high scaled alpha when market volatility is high, and for stocks with low scaled alpha when market volatility is low.

Because the survival of stocks with high idiosyncratic volatility and low expected return is determined by their betas and the market volatility jointly (see Lemmas 1 and 2), an increase in market volatility for stocks with high scaled alpha makes their survival more likely (see Propositions 4 and 7). The same is true when a decrease in market volatility occurs for stocks with low scaled alpha.

Prediction 7. When the degree of conformity pressure in the population is high, asset prices are more likely to deviate from their fundamental values, market efficiency tends to be lower, and the speed of convergence after large market shocks tends to be slower.

See Section 7.1.

Prediction 8. Asset bubbles and bursts are more likely to occur when the degree of attention to novelty in the population is high.

See Section 7.2.

Appendix E discusses potential ways to perform empirical tests on these predictions.

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[34] See Baker et al. (2011) and Frazzini and Pedersen (2014).

[35] See Ang et al. (2006, 2009).

9. Conclusion

In a cultural evolutionary model with competing investment philosophies that place different probability weights on two investment styles, we have shown that in equilibrium, the market consists of a mixed population that invests in both investment styles. This implies a wider variation of coexisting strategies than in traditional models, as exemplified by the mutual fund separation theorems deriving from versions of the Capital Asset Pricing Model (Sharpe, 1964; Merton, 1972).

The survival of investment philosophies is jointly determined by several elements, including the asset's mean return, beta, idiosyncratic volatility, and market volatility. We also derive the evolutionary equilibrium investment philosophy with respect to these return characteristics. In general, higher mean returns

[illegible]

$$\mathbb{E}[1/\ ] = \mathbb{E}\left[— \right] \approx 1 + \left(\frac{\beta^2 \beta}{\mu^3} \right) \left(\frac{\mu}{\beta} - \frac{\mu}{\beta} \right) \mathrm{Var}(\ ) + \left(\frac{\mu}{\mu^3} \right) \mathrm{Var}(\epsilon\ ) > 1.$$

Up to a second-order Taylor approximation, $\mathbb{E}[1/\ ]$ is always greater than 1, which implies that style alone is never an equilibrium. The long-run equilibrium philosophy is either purely style (with a higher scaled alpha), or a combination of both investment styles, in which case the first-order condition for in Lemma 2 reduces to:

$$0 = [\ \beta\ + (1 - \ )\beta\ ] (\beta\ - \beta\ )\ (\ ) + (1 - \ )\ (\epsilon\ ) - \ (\epsilon\ ),$$

from which the evolutionary equilibrium philosophy * can be solved. We summarize these observations as follows:

Proposition A.1. [illegible] ϵ [illegible] ϵ [illegible]

$$(\epsilon\ ) < \beta\ (\beta\ - \beta\ )\ (\ ). \tag{A.1}$$

[illegible]

$$^* = \frac{(\epsilon\ ) - \beta\ (\beta\ - \beta\ )\ (\ )}{(\epsilon\ ) + \ (\epsilon\ ) + (\beta\ - \beta\ )^2\ (\ )}. \tag{A.2}$$

It is evident from Proposition A.1 that the population tends to have only investors in style when the common component has a high volatility (( )), the safer style has a low volatility ($(\epsilon\ )$), and the riskier style has a high beta (β). In the case that the population consists of investors in both styles, the fraction of investors in style increases as the variance of the -specific component ($(\epsilon\ )$) decreases, the variance of the -specific component ($(\epsilon\ )$) increases, and the variance of the common component (( )) decreases. This is consistent with our earlier discussions indicating that risk tends to reduce the evolutionary success of a style.

When comparing the riskier style and the safer style, Proposition A.1 implies that the riskier style alone is never optimal. A certain amount of allocation in the safer style is always desirable. It also implies that allocation in the riskier style tends to increase in stable environments and decrease in volatile markets.

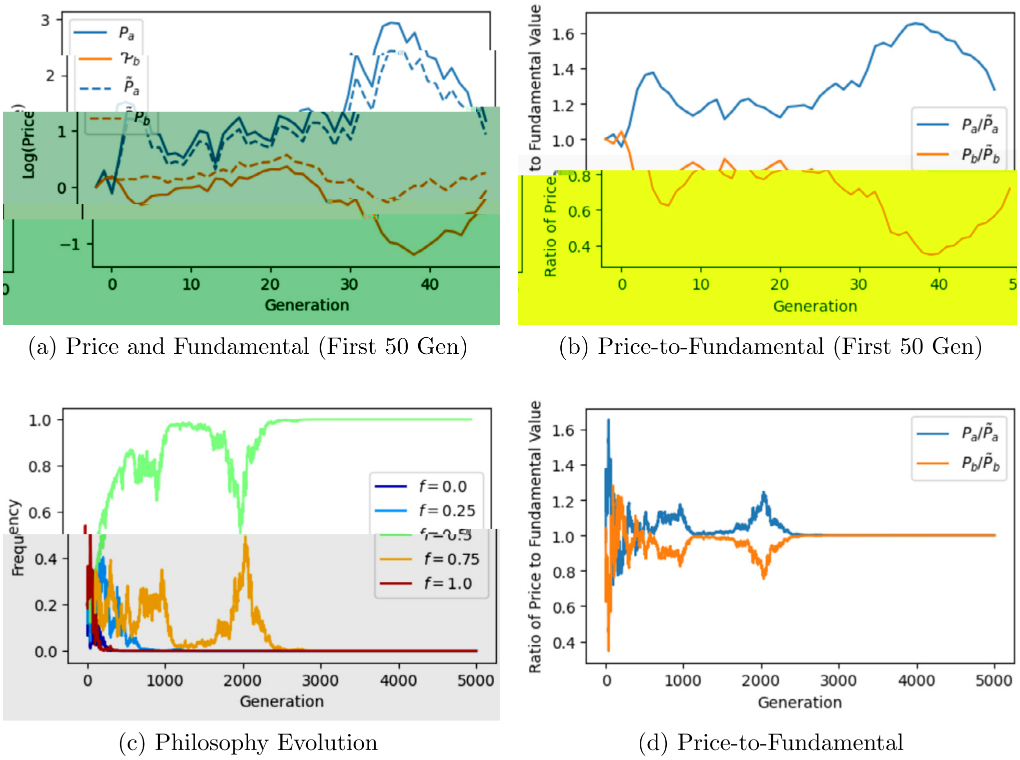
Appendix B. Additional results for market equilibrium

Table A.1 shows the equilibrium prices when different philosophies survive in equilibrium for several different values of , the elasticity of supply with respect to price deviations from the fundamental value.

We then provide additional simulation examples to demonstrate the effect in market equilibrium. We consider a market in which investment returns are given by the same specification in Equation (25) as the simulated example in the main paper. Fig. A.1 demonstrates a market in which prices are determined endogenously, with five philosophies ($= 0, 0.25, 0.5, 0.75, 1$) over 5,000 generations. Figs. A.1a–A.1b focuses on the first 50 generations, and show the (log)-endogenous price, the (log)-fundamental value, and the price-to-fundamental ratio, respectively. Prices fluctuate around the fundamental value. Style is overpriced in this period due to its high demand initially. Fig. A.1c shows the evolution of five philosophies ($= 0, 0.25, 0.5, 0.75, 1$) over 5,000 generations, in which the vertical axis denotes the frequency of each type of investor in the population. $= 1.0$ is popular for a short period of time in the very beginning, consistent with the fact that style is over-priced in Figs. A.1a–A.1b. After that, the equilibrium philosophy $^* = 0.5$ quickly dominates the population. Finally, Fig. A.1d shows the price-to-fundamental ratio over the entire course

Table A.1
A comparison between the ratio of surviving philosophies and the ratio of equilibrium prices for different levels of the elasticity of supply with respect to price deviations from the fundamental value.

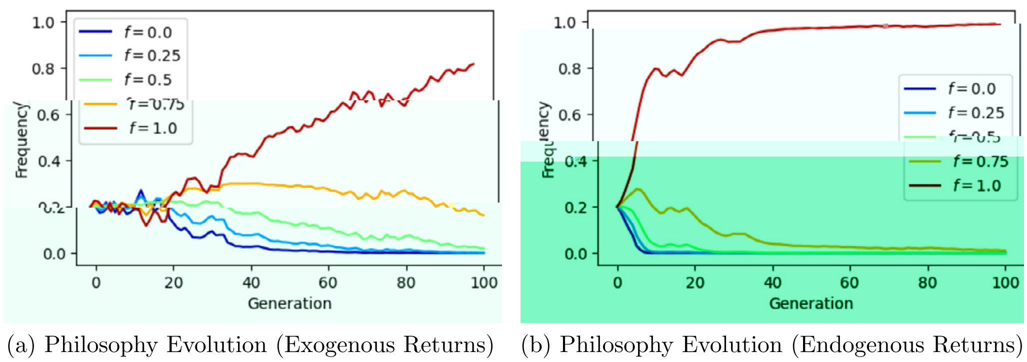
Elasticity of supply	Ratio of philosophies $_1/_2$	Ratio of prices $P_{,1}/P_{,2}$
	5	1.17
	2	1.07
10	1	1.00
	0.5	0.93
	0.2	0.85
	5	1.38
	2	1.15
5	1	1.00
	0.5	0.87
	0.2	0.72
	5	2.24
	2	1.41
2	1	1.00
	0.5	0.71
	0.2	0.45



(a) Price and Fundamental (First 50 Gen) (b) Price-to-Fundamental (First 50 Gen)

(c) Philosophy Evolution (d) Price-to-Fundamental

Fig. A.1. A demonstration of market equilibrium in which prices and returns are determined endogenously. (A.1a) and (A.1b) show the fundamental value, price, and price-to-fundamental ratio over the first 50 generations in evolution. (A.1c) and (A.1d) show the equilibrium philosophy and the price-to-fundamental ratio over 5,000 generations.



(a) Philosophy Evolution (Exogenous Returns) (b) Philosophy Evolution (Endogenous Returns)

Fig. A.2. Market equilibrium speeds up the rate of convergence. The evolution of the equilibrium philosophy * with exogenous returns (A.2a) and the equilibrium philosophy with endogenous returns (A.2b) are shown over 100 generations.

of the evolution. After an extended period of fluctuations, the ratio eventually converges to one. In reality, the market conditions are constantly changing. Instead of the long-run limit, the short-term oscillation shown here may be typical of the market.

In a slightly different simulation experiment, we increase the mean return of style so that in equilibrium $= 1$ is the dominant behavior:

$$\mu = 1.1, \quad \mu = 1, \quad \beta = 2, \quad \beta = 0.1,$$
$$\sim N(0, 0.1^2), \quad \epsilon \sim N(0, 0.3^2), \quad \epsilon \sim N(0, 0.1^2), \tag{A.3}$$

where N denotes the normal distribution. We also set $= 0.3$, $= 1.2$, and $_F = 1$.

Fig. A.2 shows that market equilibrium prices may speed up the rate of convergence, by comparing the evolution of the same five philosophies ($= 0, 0.25, 0.5, 0.75, 1$) when returns are exogenously determined by the fundamental value (Fig. A.2a), and when returns are endogenously determined by market equilibrium (Fig. A.2b). In the former, the market still contains multiple philosophies after 100 generations, while in the latter, $= 1.0$ dominates the population after around 50 generations.

This phenomenon can be understood by the expression of equilibrium returns in Equation (21). When the aggregate demand is, for example, increasing for style , market equilibrium forces further enhance the returns for that style. In this sense, market equilibrium serves as a sort of momentum for style returns, thereby helping the dominant style to dominate faster. The same mechanism is also adopted in the computer science literature for optimizing the loss function of deep neural networks.[36]

Appendix C. Generalization for multiple styles

Our main model in Section 3 considers two competing investment styles whose returns share a common factor. The simplicity of this specification allows us to derive closed-form expressions that highlight many key economic insights. However, our model can be substantially generalized to include multiple investment styles. We describe this extension here.

Consider investors who choose from investment styles (or assets), $\{1, \cdots, \}$, and this results in one of corresponding random payoffs, $(_1, \cdots,)$. Suppose each individual chooses style with probability , for $= 1, 2, \cdots,$. Let $\mathbf{p} = (_1, \cdots,)$ be the probability vector that characterizes an individual's investment philosophy. $\mathbf{p}$ satisfies the following conditions:

$$0 \leq \quad \leq 1, \quad \forall \; = 1, \cdots,$$

$$\sum_{=1} \quad = 1.$$

The style returns are determined by the following factor structure:

$$\begin{cases} _1 = \mu_1 + \beta_1 \; + \epsilon_1 \\ \cdots \\ \quad = \mu \; + \beta \; + \epsilon \; . \end{cases}$$

For simplicity, we write $\mathbf{X} = (_1, \ldots,)$ to denote the vector of all style returns. In the multinomial choice model, the population growth rate is determined by the vector $\mathbf{p}$. Therefore, it is convenient to consider the number of offspring for individual with type $= \mathbf{p}$:

$$\mathbf{p}^* = \begin{cases} (1,0,\cdots,0) & \mathbb{E}\left[\frac{2}{1}\right] < 1, \mathbb{E}\left[\frac{3}{1}\right] < 1, \cdots, \mathbb{E}\left[\frac{}{1}\right] < 1 \\ (0,1,\cdots,0) & \mathbb{E}\left[\frac{1}{2}\right] < 1, \mathbb{E}\left[\frac{3}{2}\right] < 1, \cdots, \mathbb{E}\left[\frac{}{2}\right] < 1 \\ \cdots \\ (0,0,\cdots,1) & \mathbb{E}\left[\frac{1}{}\right] < 1, \mathbb{E}\left[\frac{2}{}\right] < 1, \cdots, \mathbb{E}\left[\frac{-1}{}\right] < 1 \end{cases} \tag{A.6}$$

$\mathbf{p}^* = ({}^*_1, \cdots, {}^*, 0, \cdots, 0)$. $\mathbf{p}^*$

$$\mathbb{E}\left[\frac{1}{{}^*_1 \; {}_1 + \cdots + {}^*}\right] = \cdots = \mathbb{E}\left[\frac{}{{}^*_1 \; {}_1 + \cdots + {}^*}\right] = 1, \tag{A.7}$$

$\mathbf{p}^*$

$$\begin{cases} \mathbb{E}\left[\frac{+1}{{}^*_1 \; {}_1 + \cdots + {}^*}\right] < 1 \\ \cdots \\ \mathbb{E}\left[\frac{}{{}^*_1 \; {}_1 + \cdots + {}^*}\right] < 1. \end{cases} \tag{A.8}$$

Proposition A.3 asserts that $\mathbf{p}^* = ({}^*_1, \cdots, {}^*, 0, \cdots, 0)$ is optimal if and only if the expectation of any irrelevant style divided by the optimal combination of styles is less than 1, and any style in the optimal combination divided by the optimal combination is equal to 1.

Proposition A.3 generalizes Proposition 1 in our main model. Comparative statics results with respect to mean return, beta, and volatilities can therefore be carried out in principle. In particular, in the first cases of Equation (A.6) when there is a single dominant style, the conditions are very similar to those in Proposition 1.[37] Therefore, we have the following comparative statics for style 1-investors, without loss of generality, which generalizes our results in Section 4.

Proposition A.4 1-

(i) 1, μ_1,

(ii) , μ , $= 2, 3, \ldots,$

(iii) , β , $= 2, 3, \ldots,$

(iv) 1 , β_1, 1'

$$\frac{\mu_1/\beta_1}{\max_{\neq 1}\{\mu / \beta \}} > 2;$$

(v) 1 , β_1, 1'

$$\frac{\mu_1/\beta_1}{\min_{\neq 1}\{\mu / \beta \}} < 2;$$

(vi) 1, (ϵ_1),

(vii) , ( ), 1'

$$\frac{\mu_1}{\beta_1} > \max_{\neq 1} \frac{\mu}{\beta};$$

(viii) , ( ), 1'

$$\frac{\mu_1}{\beta_1} < \min_{\neq 1} \frac{\mu}{\beta}.$$

( ) ( ) , ϵ ϵ .

[37] However, when the evolutionary equilibrium philosophy involves a mix of multiple investment styles, the condition in Equation (A.7) defines $\mathbf{p}^*$, but the analytic comparative statics become intractable.

[illegible]

Propositions A.3–A.4 together show that our results on co-existence of investment styles and their comparative statics in Section 4 hold true in the multi-style setting.

Appendix D. Additional discussions on psychological bias

[illegible]

We first consider population dynamics with conformist preference as specified in Equation (24). By a similar derivation as in Equation (3), the population size of type- investors in period is:

$$= \prod_{=1} [\quad + (1 - \ ) \quad] \exp \left[\tau (\ - \lambda_{-1})^2 \right]$$

$$= \exp \left\{ \sum_{=1} \log [\quad + (1 - \ ) \quad] + \tau \sum_{=1} (\ - \lambda_{-1})^2 \right\}.$$

Taking the logarithm of the number of offspring, we have:

$$\lim_{\to \infty} \frac{1}{} \log \ = \mathbb{E}[\log (\quad + (1 - \ ) \quad)] + \tau \lim_{\to \infty} \frac{1}{} \sum_{=1} (\ - \lambda_{-1})^2, \tag{A.9}$$

where the first term is simply the log-geometric average growth rate of the population without conformity pressure, $\alpha(\ )$, in Equations (4). From Equations (24) and (A.9), we can see that the magnitude of the conformity pressure τ acts roughly as a multiplicative factor in the fitness, or an additive factor in the population growth rate.[38]

The case of attention to novelty as specified in Equation (27) is similar to the case of conformity. The logarithm of the population size is:

$$\lim_{\to \infty} \frac{1}{} \log \ = \mathbb{E}[\log (\quad + (1 - \ ) \quad)] + \rho \lim_{\to \infty} \frac{1}{} \sum_{=1} (1 - \ _{-1}), \tag{A.10}$$

where the first term is again the log-geometric average growth rate of the population without attention to novelty, $\alpha(\ )$, in Equation (4).[39] Suppose a long time has passed, and that a philosophy almost dominates the population. The second term in Equation (A.10) is close to 0 for that philosophy, while other philosophies receive a fitness boost due to the attention to novelty, and may tend to outgrow the currently popular philosophy. Therefore, it is hard for any single philosophy to dominate in the long run.

[illegible]

When the degree of conformist preference is extreme, we have seen that the convergence to the long-run equilibrium investment philosophy can be greatly delayed (see Fig. 2). This is not surprising, as the “wisdom of crowds” only works under the assumption that individuals have different information sources and relatively independent decision-making processes. If this condition is violated, the “effective population size” (to borrow a term from population genetics) is greatly reduced, and crowds may have little wisdom.

On the other extreme, when the degree of attention to novelty is high, investment philosophies that work well in the current environment have a weaker influence on the adoption of philosophies in the future. Investors no longer use the information from past returns embedded in the population frequencies. As a result, no one benefits from the “wisdom of crowds”, which can lead to bubbles and bursts (see Fig. 3).

In practice, an intermediate amount of social learning is probably most desirable from the perspective of adopting the fittest philosophy in the current environment. For example, studies on interactions between financial traders have documented a large range of rates of idea flow, from isolated individual traders at one end to traders trapped in an

With the collection of “Big Data” in the digital era, another promising financial data source is social media.[41] Modern digital data includes information about call records, credit card transactions, and social network usage, among other recorded interactions. This data is particularly useful to measure social transmission effects such as conformist preference and attention to novelty in our model.

Empirical tests for the effects of attention to novelty are possible using proxies for attention that have been applied in the empirical finance literature (see, for example, Barber and Odean (2007), Da et al. (2011), and Li and Yu (2012)). Henderson and Pearson (2011) find evidence that firms issue certain retail structured equity products with negative expected returns, potentially shrouding some aspects of securities innovation or introducing complexity to attract attention, therefore exploiting uninformed investors. This suggests that some investors do invest based on attention to novelty even if the financial security might not deliver desirable returns, which is consistent with our assumptions.

Appendix F. Proofs

Proof of Proposition 1. This is first proved by Brennan and Lo (2011) and we reproduce the proof here for completeness. This follows from the first and second derivatives of Equation (4). Because the second derivative is strictly negative, there is exactly one maximum value obtained in the interval $[0,1]$. The values of the first-order derivative of $\alpha(\ )$ at the endpoints are given by:

$$\alpha'(0) \;=\; \mathbb{E}[\ \ /\ \ ] - 1 \;,\quad \alpha'(1) \;=\; 1 - \mathbb{E}[\ \ /\ \ ]\,.$$

If both are positive or both are negative, then $\alpha(\ )$ increases or decreases, respectively, throughout the interval and the maximum value is attained at $\ =1$ or $\ =0$, respectively. Otherwise, $\ =\ ^*$ is the unique point in the interval for which $\alpha'(\ )=0$, where $\ ^*$ is defined in Equation (6), and it is at this point that $\alpha(\ )$ attains its maximum value. The expression in Equation (5) summarizes the results of these observations for the various possible values of $\mathbb{E}[\ \ /\ \ ]$ and $\mathbb{E}[\ \ /\ \ ]$. Note that the case $\mathbb{E}[\ \ /\ \ ] \le 1$ and $\mathbb{E}[\ \ /\ \ ] \le 1$ is not considered because this set of inequalities implies that $\alpha'(0) \le 0$ and $\alpha'(1) \ge 0$, which is impossible since $\alpha''(\ )$ is strictly negative. □

Proof of Proposition 2. $\mathbb{E}[1/\ ]$ as given in Equation (8) is a decreasing function of μ and an increasing function of μ. □

Proof of Lemma 1. According to the discussion leading to Lemma 1, calculations of second-order derivatives of $(\ ,\epsilon\ ,\epsilon\ )$ suffice. For simplicity, we use $(0,0,0)$ to represent $\ =\epsilon\ =\epsilon\ =0$.

$$\frac{\partial}{\partial} = \frac{\beta\,(\mu\ +\beta\ \ +\epsilon\ ) - \beta\,(\mu\ +\beta\ \ +\epsilon\ )}{(\mu\ +\beta\ \ +\epsilon\ )^2} = \frac{\beta\ \mu\ -\beta\ \mu\ +\beta\ \epsilon\ -\beta\ \epsilon}{(\mu\ +\beta\ \ +\epsilon\ )^2}$$

$$\frac{\partial^2}{\partial\ ^2} = \frac{-2\beta\,(\beta\ \mu\ -\beta\ \mu\ +\beta\ \epsilon\ -\beta\ \epsilon\ )}{(\mu\ +\beta\ \ +\epsilon\ )^3} \overset{(0,0,0)}{=\!=} \frac{2\beta\,(\beta\ \mu\ -\beta\ \mu\ )}{\mu^3}$$

$$\frac{\partial}{\partial\epsilon} = \frac{1}{\mu\ +\beta\ \ +\epsilon}, \qquad \frac{\partial^2}{\partial\epsilon^2} = 0$$

$$\frac{\partial}{\partial\epsilon} = -\frac{\mu\ +\beta\ \ +\epsilon}{(\mu\ +\beta\ \ +\epsilon\ )^2}$$

$$\frac{\partial^2}{\partial\epsilon^2} = \frac{2(\mu\ +\beta\ \ +\epsilon\ )}{(\mu\ +\beta\ \ +\epsilon\ )^3} \overset{(0,0,0)}{=\!=} \frac{2\mu}{\mu^3}.$$

Therefore,

$$\mathbb{E}[\ ] \approx \frac{\mu}{\mu} + \frac{\beta\,(\beta\ \mu\ -\beta\ \mu\ )}{\mu^3}\mathbb{E}[\ ^2] + \frac{\mu}{\mu^3}\mathbb{E}[\epsilon^2] = \frac{\mu}{\mu} + \frac{\beta\ \beta^2}{\mu^3}\left(\frac{\mu}{\beta} - \frac{\mu}{\beta}\right)\mathrm{Var}(\ ) + \frac{\mu}{\mu^3}\mathrm{Var}(\epsilon\ ), \tag{A.11}$$

which completes the proof of the first part. The approximation for $\mathbb{E}[1/\ ]$ follows from similar calculations. □

Proof of Proposition 3. According to Lemma 1, $\mathbb{E}[1/\ ]$ is a decreasing function of β; it is a quadratic function of β and therefore turns at its vertex. □

Proof of Proposition 4. It follows directly from Lemma 1. □

Proof of Proposition 5. The first-order condition as given in Equation (9) is a decreasing function of $\ $, an increasing function of μ, and a decreasing function of μ. Therefore, as μ increases, the solution $\ ^*$ has to increase. Similarly, as μ decreases, the solution $\ ^*$ has to increase. □

[41] Some examples of such social media services include SeekingAlpha, StockTwits (used in Cookson and Niessner (2020) and Argarwal et al. (2018)), eToro (used in Altshuler et al. (2012), Pan et al. (2012), and Pentland (2015)), and an unnamed European social trading platform used in Ammann and Schaub (2021).

$$\mathbb{E}[\ ] = \mathbb{E}\left[\frac{\psi(\ )}{\psi(\ )}\right] \approx \frac{\psi(\mu\ )}{\psi(\mu\ )} + \frac{1}{2}\left(\frac{\partial^2\ _0}{\partial\ ^2}\mathrm{Var}(\ ) + \frac{\partial^2\ _0}{\partial\epsilon^2}\mathrm{Var}(\epsilon\ ) + \frac{\partial^2\ _0}{\partial\epsilon^2}\mathrm{Var}(\epsilon\ )\right).$$

We then calculate second-order derivatives of $(\ ,\epsilon\ ,\epsilon\ )$. For simplicity, we use $(0,0,0)$ to represent $\ =\epsilon\ =\epsilon\ =0$.

$$\frac{\partial}{\partial\ } = \beta\ \psi'(\ )\psi(\ ) - \beta\ \psi'($$

$$\frac{\partial^2 F}{\partial \epsilon^2} \overset{(0,0,0)}{=\!=\!=} \frac{\psi''(\mu\ )\psi(\mu\ )\left[\ \psi(\mu\ ) + (1 - \ )\psi(\mu\ )\right] - 2\ \left(\psi'(\mu\ )\right)^2 \psi(\mu\ )}{\left[\ \psi(\mu\ ) + (1 - \ )\psi(\mu\ )\right]^2} > 0$$

$$\Longleftrightarrow \psi''(\mu\ ) > \frac{2\ \left(\psi'(\mu\ )\right)^2}{\ \psi(\mu\ ) + (1 - \ )\psi(\mu\ )}.$$

Similarly,

$$\frac{\partial^2 F}{\partial \epsilon^2} \overset{(0,0,0)}{=\!=\!=} \frac{-\psi(\mu\ )\psi''(\mu\ )\left[\ \psi(\mu\ ) + (1 - \ )\psi(\mu\ )\right] + 2(1 - \ )\psi(\mu\ )\left(\psi'(\mu\ )\right)^2}{\left[\ \psi(\mu\ ) + (1 - \ )\psi(\mu\ )\right]^2} < 0$$

$\Longleftrightarrow \psi''(\mu$ .830

which further leads to:

$$\mathbb{E}\left[\frac{({}_1 - {}^*_1)\ {}_1 + \cdots + (\ - \ {}^*)}{{}^*_1\ {}_1 + \cdots + \ {}^*}\right] \leq 0, \text{ for any } \mathbf{p} = ({}_1, \cdots, \ )$$

$$\Longrightarrow \mathbb{E}\left[\frac{{}_1\ {}_1 + \cdots + }{{}^*_1\ {}_1 + \cdots + \ {}^*}\right] \leq 1, \text{ for any } \mathbf{p} = ({}_1, \cdots, \ )$$

which completes the proof. □

Proof of Proposition A.3. The first conditions in Equation (A.6) follow directly from Proposition A.2. As for the last case, note that ${}_1 = 1 - {}_2 - \cdots -$ and we can write $\mu(\cdot)$ as a function of $({}_2, \cdots, \ )$. Therefore $\mathbf{p}^*$ is given by the

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