



JEL C a c a t G11 · G12 · D03 · D11

1 I t r d c t

The allocation of wealth among alternative assets is one of an individual's most important financial decisions. The groundbreaking work of [Markowitz \(1952\)](#) in mean-variance theory, used to analyze asset allocation, has remained the cornerstone of modern portfolio theory. This has led to numerous breakthroughs in financial economics, including the famous Capital Asset Pricing Model ([Sharpe 1964](#); [Lintner 1965a, b](#); [Treynor 1965](#); [Mossin 1966](#)). The influence of this paradigm goes far beyond academia, however. For example, it has become an integral part of modern investment management practice ([Reilly and Brown 2011](#)).

More recently, economists have also adopted the use of evolutionary principles to understand economic behavior, leading to the development of evolutionary game theory ([Marschall and Smith 1982](#)); the evolutionary implications of probability matching ([Cooper and Kaplan 1982](#)), group selection ([Zhang et al. 2014a](#)), and cooperation and altruism ([Alexander 1974](#); [Hirshleifer 1977, 1978](#)); and the process of selection of firms ([Luo 1995](#)) and traders ([Blume and Easley 1992](#); [Kogan et al. 2006a](#); [Hirshleifer and Teoh 2009](#)). The Adaptive Markets Hypothesis ([Lo 2004, 2017](#)) provides economics with a more general evolutionary perspective, reconciling economic theories based on the Efficient Markets Hypothesis with behavioral economics. Under this hypothesis, the neoclassical models of rational behavior coexist with behavioral models, and what had previously been cited as counterexamples to rationality—loss aversion, overconfidence, overreaction, and other behavioral biases—become consistent with an evolutionary model of human behavior.

The evolutionary perspective brings new insights to economics from beyond the traditional neoclassical realm, helping to reconcile inconsistencies in behavior between *Homo economicus* and *Homo sapiens* ([Kahneman and Tversky 1979](#); [Brennan and Lo 2011](#)). In particular, evolutionary models of behavior provide important insights into the biological origin of time preference and utility functions ([Rogers 1994](#); [Waldman 1994](#); [Samuelson 2001](#); [Zhang et al. 2014b](#)), in fact justifying their existence, and allow us to derive conditions about their functional form ([Hansson and Stuart 1990](#); [Robson 1996, 2001a](#)) (see [Robson 2001b](#); [Robson and Samuelson 2009](#) for comprehensive reviews of this literature). In addition, the experimental evolution of biological organisms has been suggested as a novel approach to understanding economic preferences, given that it allows the empirical study of preferences by placing organisms in specifically designed environments ([Burnham et al. 2015](#)).

The evolutionary approach to investing is closely related to optimal portfolio growth theory, as explored by [Kell \(1956\)](#), [Hakansson \(1970\)](#), [Thorp \(1971\)](#), [Algoet and Cover \(1988\)](#), [Browne and Whitt \(1996\)](#), and [Aurell et al. \(2000\)](#), among others. While the evolutionary framework tends to focus on the long-term performance of a strategy, investors are also concerned with the short to medium term ([Browne 1999](#)). Mopic investor behavior has been documented in both theoretical and empirical studies ([Strot 1955](#); [Strotman 1983](#); [Thaler et al. 1997](#); [Bushee 1998](#)). Since much of the field of population genetics focuses on short-term competition between different

types of individuals, population geneticists have applied their ideas to portfolio theory (Frank 1990, 2011; Orr 2017), in some cases considering the maximization of one-period expected wealth.

The market dynamics of investment strategies under evolutionary selection have been explored under the assumption that investors will try to maximize absolute wealth (Evstigneev et al. 2002; Amir et al. 2005; Hens and Schenk-Hopp 2005; Evstigneev et al. 2006). Some studies have found that individual investors with more accurate beliefs will accumulate more wealth, and thus dominate the economy (Sandroni 2000, 2005), while others have argued that wealth dynamics need not lead to rules that maximize expected utility using rational expectations (Blume and Easley 1992), and that investors with incorrect beliefs may drive out those with correct beliefs (Blume and Easley 2006). Research on the performance of rational versus irrational traders has also adopted evolutionary ideas; for example, it has been shown that irrational traders can survive in the long run, resulting in prices that diverge from fundamental values (Long et al. 1990, 1991; Biais and Shadrur 2000; Hirshleifer and Luo 2001; Hirshleifer et al. 2006; Kogan et al. 2006b; Yan 2008).

Traditional portfolio growth theory has focused on absolute wealth and the Kell criterion (Kell 1956; Thorp 1971). Instead of studying the growth of absolute wealth, however, we will consider the relative wealth in the spirit of Orr (2017). Relative wealth or income has been discussed in a number of studies (Robson 1992; Bakshi and Chen 1996; Corneo and Jeanne 1997; Hens and Schenk-Hopp 2005; Frank 2011), and the behavioral economics literature provides voluminous evidence that investors sometimes assess their performance relative to a reference group (Frank 1985; Clark and Oswald 1996; Clark et al. 2008). It is particularly important to understand the consequences of investment decisions in a setting where relative wealth is the standard, not absolute wealth. As Burnham et al. (2015) pointed out, if people are envious but caring about relative wealth, then free trade may make all parties richer, but may cause envious people to be less happy. If economics misunderstands human nature, then free trade may simultaneously increase wealth and unhappiness.

In this paper, we compare the implications of maximizing relative wealth to maximizing absolute wealth over both short-term and long-term investment horizons. We use ideas from Orr (2017), and compare his results to an extension of the binary choice model of Brennan and Lo (2011). We consider two assets in a discrete-time model, and an investor who allocates her wealth between the two assets. Rather than maximizing her absolute wealth, the investor maximizes her wealth relative to another investor with a fixed behavior. We consider the cases of one time period, multiple periods, and an infinite time horizon. We then ask the question: what is the optimal behavior for an investor as a function of the environment, given that the environment consists of the asset returns and the behavior of the other participants? In our approach, we define relative wealth as a proportion of the total wealth, which corresponds closely to the allele frequency in population genetics. This analogy acts as a bridge to earlier literature on the relevance of relative wealth to behavior. While some of our results will be familiar to population geneticists, they do not appear to be widely known in a financial context. For completeness, we derive them from first principles in this new context.

Our approach leads to several interesting conclusions about the Kell criterion. We show that it is the optimal behavior if the investor maximizes her absolute wealth in the

case of an infinite horizon (see also [Brennan and Lo 2011](#)). In the case that the investor maximizes her relative wealth, we identify the conditions under which it is optimal, and the conditions under which the investor should deviate from it. The investor's initial relative wealth—which represents the investor's market power—plays a critical role. Moreover, the dominant investor's optimal behavior is different from the minorant investor's optimal behavior.

In Sect. 2 of this paper, we consider a two-asset model in which investors maximize their absolute wealth. It is shown that the long-run optimal behavior is equivalent to the behavior implied by the Kell criterion. Section 3 extends the binary choice model, and considers in a non-game-theoretic framework the case of two investors who maximize their wealth relative to the population, given the other investor's behavior. The Kell criterion emerges as a special case under certain environmental conditions. Section 4 provides a numerical example to illustrate the theoretical results. Section 5 discusses several implications which can be tested through experimental evolutionary techniques. We end with a discussion in Sect. 6, and provide proofs in Appendix A.

2 Maximizing absolute wealth: the Kelly criterion

Consider two assets, a and b , in a discrete-time model, each generating gross returns $X_a \in (0, \infty)$ and $X_b \in (0, \infty)$ in one period. For example, asset a can be a risky asset whereas asset b can be the riskless asset. In this case, $X_a \in (0, \infty)$ and $X_b = 1 + r$, where r is the risk-free interest rate. In general, $(X_{a,t}, X_{b,t})$ are IID over time $t = 1, 2, \dots$, and are described by the probability distribution function $\Phi(X_a, X_b)$.

Consider an investor who allocates $f \in [0, 1]$ of her wealth in asset a and $1 - f$ in asset b . We will refer to f as the investor's behavior henceforth. We assume that:

Assumption 1 (X_a, X_b) and $\log(fX_a + (1 - f)X_b)$ have finite moments up to order 2 for all $f \in [0, 1]$.

Note that Assumption 1 guarantees that the gross return of an investment portfolio is positive. In other words, the investor cannot lose more than what she has. This is made possible by assuming that X_a and X_b are positive and f is between 0 and 1. In other words, the investor only allocates her money between two assets, and no short-selling or leverage is allowed.¹

Let n_t^f be the total wealth of investor f in period t . To simplify notation, let $\omega_t^f = fX_{a,t} + (1 - f)X_{b,t}$ be the gross return of investor f 's portfolio in period t . With these notational conventions in mind, the portfolio growth from period $t - 1$ to period t is:

$$n_t^f = n_{t-1}^f (fX_{a,t} + (1 - f)X_{b,t}) = n_{t-1}^f \omega_t^f.$$

¹ One could relax this assumption by allowing short-selling and leverage, which corresponds to $f < 0$ or $f > 0$. However, f still needs to be restricted such that $fX_a + (1 - f)X_b$ is always positive. This does not change our results in an essential way, but it will complicate the presentation of some results mathematically. Therefore, we stick to the simple assumption that $f \in [0, 1]$ as in [Brennan and Lo \(2011\)](#).

Through backward recursion, the total wealth of investor f in period T is given by

$$n_T^f = \prod_{t=1}^T \omega_t^f = \exp \left(\sum_{t=1}^T \log \omega_t^f \right).$$

Taking the logarithm of wealth and applying Kolmogorov's law of large numbers, we have:

$$\frac{1}{T} \log n_T^f = \frac{1}{T} \sum_{t=1}^T \log \omega_t^f \xrightarrow{P} \mathbb{E} [\log \omega_t^f] = \mathbb{E} [\log (fX_a + (1-f)X_b)], \quad (1)$$

as T increases without bound, where \xrightarrow{P} in (1) denotes convergence in probability. We have assumed that $n_0^f = 1$ without loss of generality.

The expression (1) is simply the expectation of the log-geometric-average growth rate of investor f 's wealth, and we will call it $\mu(f)$ henceforth:

$$\mu(f) = \mathbb{E} [\log (fX_a + (1-f)X_b)]. \quad (2)$$

The optimal f that maximizes (2) is given by

Proposition 1 *The optimal allocation f^{Kelly} that maximizes investor f 's absolute wealth as T increases without bound is*

$$f^{Kelly} = \begin{cases} 1 & \text{if } \mathbb{E}[X_a/X_b] > 1 \text{ and } \mathbb{E}[X_b/X_a] < 1, \\ \text{solution to (4)} & \text{if } \mathbb{E}[X_a/X_b] \geq 1 \text{ and } \mathbb{E}[X_b/X_a] \geq 1, \\ 0 & \text{if } \mathbb{E}[X_a/X_b] < 1 \text{ and } \mathbb{E}[X_b/X_a] > 1, \end{cases} \quad (3)$$

where f^{Kelly} is defined implicitly in the second case of (3) by:

$$\mathbb{E} \left[\frac{X_a - X_b}{f^{Kelly} X_a + (1 - f^{Kelly}) X_b} \right] = 0. \quad (4)$$

The optimal allocation given in Proposition 1 coincides with the Kelly criterion (Kelly 1956; Thorp 1971) in probability theory and the portfolio choice literature. To emphasize this connection, we refer to this optimal allocation as the Kelly criterion henceforth. As we will see, in the case of maximizing an individual's relative wealth, the Kelly criterion plays a key role as a reference strategy.

In portfolio theory, the Kelly criterion is used to determine the optimal size of a series of bets in the long run. Although this strategy's promise of doing better than any other strategy in the long run seems compelling, some researchers have argued against it, principally because the specific investing constraints of an individual may override

3 Maximizing relative wealth

In this section, we consider two investors. The first investor allocates $f \in [0, 1]$ of her wealth in asset a and $1 - f$ in asset b . The second investor allocates $g \in [0, 1]$ of his wealth in asset a and $1 - g$ in asset b . Investor f 's objective is to maximize the proportion of her wealth relative to the total wealth in the population, which we define as investor f 's relative wealth. Note that we use f and g to mean the proportion of wealth and as a label for the investor, to simplify notation.

Here we can introduce a concept taken from evolutionary theory. In population genetics, the metric for natural selection is the expected reproduction of a genotype divided by the average reproduction of the population, i.e., the relative reproduction, analogous to investor f 's relative wealth. Our consideration of the relative wealth rather than the absolute wealth naturally unlocks existing tools and ideas from population genetics for us.

In the case of maximizing relative wealth, the initial wealth plays an important role in the optimal allocation. Let $\lambda \in (0, 1)$ be the relative initial wealth of investor f :

$$\lambda = \frac{n_0^f}{n_0^f + n_0^g}.$$

Let q_t^f be the relative wealth of investor f in subsequent periods $t = 1, 2, \dots$. q_t^f and q_t^g are defined similarly:

$$q_t^f = \frac{n_t^f}{n_t^f + n_t^g} = \frac{1}{1 + n_t^g/n_t^f},$$

$$q_t^g = 1 - q_t^f.$$

It is obvious that the ratio n_t^g/n_t^f is sufficient to determine the relative wealth q_t^f . Let R_T^f be the T -period average log-relative-growth:

$$R_T^f = \frac{1}{T} \log \frac{\prod_{t=1}^T \omega_t^g}{\prod_{t=1}^T \omega_t^f} = \frac{1}{T} \sum_{t=1}^T \log \frac{\omega_t^g}{\omega_t^f}. \tag{5}$$

Then we can write the relative wealth in period T as:

$$q_T^f = \frac{1}{1 + \frac{n_T^g}{n_T^f}} = \frac{1}{1 + \frac{(1-\lambda) \prod_{t=1}^T \omega_t^g}{\lambda \prod_{t=1}^T \omega_t^f}} = \frac{1}{1 + \frac{1-\lambda}{\lambda} \exp\left(T R_T^f\right)}. \tag{6}$$

Analogous to Eqs. (5)–(6) are well known in the population genetics literature, used when the fitnesses of genotypes are assumed to vary randomly through time. (For reviews of this literature, see [Felsenstein 1976](#) and [Gillespie 1991](#), Chap. 4.)

3.1 Optimal investment

We first consider a myopic investor, who maximises her expected relative wealth in the first period. By (6), the expectation of q_1^f is:

$$\mathbb{E}\left[q_1^f\right] = \mathbb{E}\left[\frac{1}{1 + \frac{1-\lambda}{\lambda} \frac{\omega^g}{\omega^f}}\right].$$

Here we have dropped the subscripts in ω_1^f and ω_1^g , and instead simply use ω^f and ω^g , because there is only one period to consider.

Given investor g , we denote f_1^* as investor f 's optimal allocation that maximises $\mathbb{E}[q_1^f]$. There is no general formula to compute $\mathbb{E}[q_1^f]$ because it involves the expectation of the ratio of random variables. Population geneticists sometimes use diffusion approximations to estimate similar quantities, for example, the change in allele frequency (Gillespie 1977; Frank and Slatkin 1990; Frank 2011), which are essentially linear approximations of the nonlinear quantity using the Taylor series. The diffusion approximation is also used by Orr (2017) in a similar model for relative wealth.

Without the diffusion approximation, one can still characterise f

than investor g . Similarly, when investor g takes a position that is more conservative than the Kelly criterion, investor f should never be even more conservative than investor g .

It is interesting to compare the optimal behavior f_1^* with the Kelly criterion f^{Kelly} , which is provided in the next proposition. It shows that when g is not far from the Kelly criterion, the relationship between f_1^* and f^{Kelly} depends on the initial relative wealth of investor f .

Proposition 4 *If investor f is the dominant investor ($\lambda > \frac{1}{2}$), then she should be locally more/less risky than Kelly in the same way as investor g : for small $\epsilon > 0$,*

$$\begin{aligned} g = f^{Kelly} - \epsilon &\Rightarrow f_1^* < f^{Kelly}, \\ g = f^{Kelly} + \epsilon &\Rightarrow f_1^* > f^{Kelly}. \end{aligned}$$

If investor f is the minorant investor ($\lambda < \frac{1}{2}$), then she should be locally more/less risky than Kelly in the opposite way as investor g : for small $\epsilon > 0$,

$$\begin{aligned} g = f^{Kelly} - \epsilon &\Rightarrow f_1^* > f^{Kelly}, \\ g = f^{Kelly} + \epsilon &\Rightarrow f_1^* < f^{Kelly}. \end{aligned}$$

If investor f starts with the same amount of wealth as investor g ($\lambda = \frac{1}{2}$), then she should be locally Kelly:

$$g \approx f^{Kelly} \Rightarrow f_1^* \approx f^{Kelly}.$$

Note that when g is far from the Kelly criterion, the conclusions in Proposition 4 may not necessarily hold. Section 4 provides a numerical example (see Fig. 1b) where investor f is the minorant investor ($\lambda < \frac{1}{2}$) and $g \ll f^{Kelly}$, but $f_1^* < f^{Kelly}$. However, Orr (2017) has shown that these results are still approximately true for an g up to a diffusion approximation, which is consistent with the numerical results for maximizing one-period relative wealth in Fig. 1a. We will provide more discussion on this point in Sect. 4.

3.2 Method

The previous results are based on maximizing the expected relative wealth in period 1: $\mathbb{E}[q_1^f]$. To generalize these results to maximizing expected relative wealth in period T : $\mathbb{E}[q_T^f]$, we have:

Proposition 5 *The optimal behavior of investor f that maximizes expected relative wealth in the T th period is given by:*

$$f_T^* = \begin{cases} 1 & \text{if } \mathbb{E} \left[\frac{\exp(TR_T^1) \left(T - \sum_{t=1}^T \frac{X_{bt}}{X_{at}} \right)}{\left(1 + \frac{1-\lambda}{\lambda} \exp(TR_T^1) \right)^2} \right] > 0, \\ \text{solution to (10)} & \text{if } \mathbb{E} \left[\frac{\exp(TR_T^1) \left(T - \sum_{t=1}^T \frac{X_{bt}}{X_{at}} \right)}{\left(1 + \frac{1-\lambda}{\lambda} \exp(TR_T^1) \right)^2} \right] \leq 0 \text{ and } \mathbb{E} \left[\frac{\exp(TR_T^0) \left(\sum_{t=1}^T \frac{X_{at}}{X_{bt}} - T \right)}{\left(1 + \frac{1-\lambda}{\lambda} \exp(TR_T^0) \right)^2} \right] \geq 0, \\ 0 & \text{if } \mathbb{E} \left[\frac{\exp(TR_T^0) \left(\sum_{t=1}^T \frac{X_{at}}{X_{bt}} - T \right)}{\left(1 + \frac{1-\lambda}{\lambda} \exp(TR_T^0) \right)^2} \right] < 0, \end{cases} \quad (9)$$

where f_1^* is defined implicitly in the second case of (9) by:

$$\mathbb{E} \left[\exp(TR_T^f) \sum_{t=1}^T \frac{X_{at} - X_{bt}}{fX_{at} + (1-f)X_{bt}} \right]$$

of Sect. 4 show that the condition that g is near the Kell criterion is essential (see Fig. 2a).

3.3 Investor

Recall from (5) that the T -period average log-relative-growth R_T^f is given by :

$$R_T^f = \frac{1}{T} \sum_{t=1}^T \log \omega_t^g - \frac{1}{T} \sum_{t=1}^T \log \omega_t^f \xrightarrow{P} \mu(g) - \mu(f), \tag{11}$$

as T increases without bound. It is therefore easy to see from (6) that:

Proposition 7 *As T increases without bound, the relative wealth of investor f converges in probability to a constant:*

$$q_T^f \xrightarrow{P} \begin{cases} 0 & \text{if } \mu(f) < \mu(g), \\ \lambda & \text{if } \mu(f) = \mu(g), \\ 1 & \text{if } \mu(f) > \mu(g). \end{cases} \tag{12}$$

Proposition 7 is consistent with well-known results in the population genetics literature (see Gillespie 1973, for example) as well as in the behavioral finance literature, as in Brennan and Lo (2011). It asserts that investor f 's relative wealth will converge to 1 as long as its log-geometric-average growth rate $\mu(f)$ is greater than investor g 's. This implies that when T increases without bound, there are multiple behaviors that are all optimal in the following sense:

$$\begin{aligned} \arg \max_f \lim_{T \rightarrow \infty} q_T^f &= \arg \max_f \mathbb{E} \left[\lim_{T \rightarrow \infty} q_T^f \right] \\ &= \arg \max_f \lim_{T \rightarrow \infty} \mathbb{E} \left[q_T^f \right] = \{f : \mu(f) > \mu(g)\}. \end{aligned}$$

Note that the above equality uses the dominant convergence theorem (q_T^f is always bounded) to switch the limit and the expectation operator.

However, this is not equivalent to the limit of the optimal behavior f_T^* as T increases without bound because one cannot switch the operator, $\arg \max$ and \lim in general, and

$$\arg \max_f \lim_{T \rightarrow \infty} \mathbb{E} \left[q_T^f \right] \neq \lim_{T \rightarrow \infty} \arg \max_f \mathbb{E} \left[q_T^f \right].$$

In fact, Sect. 4 provides such an example.

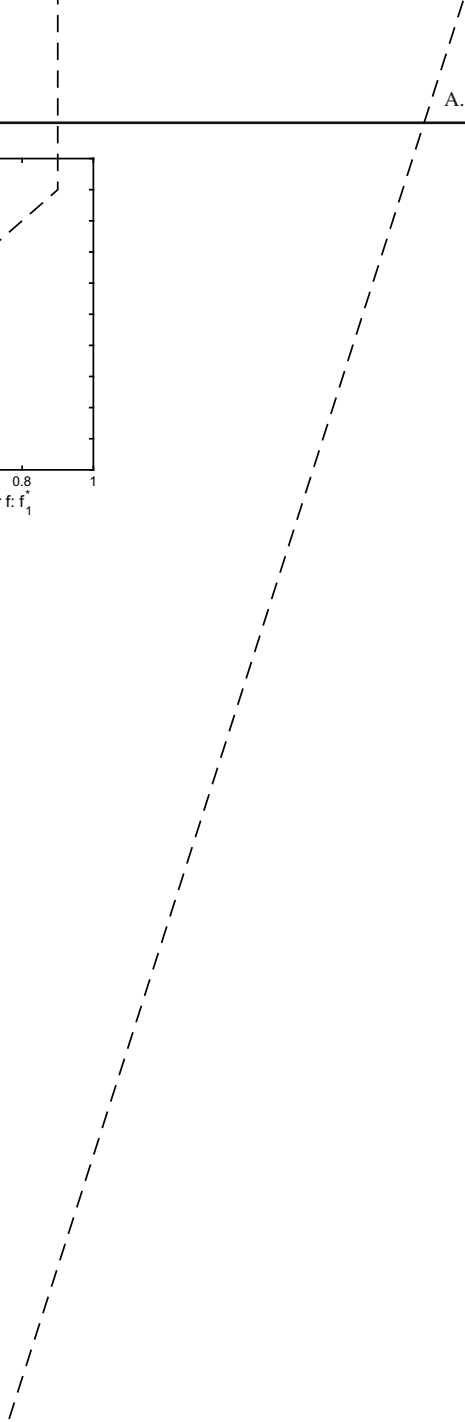
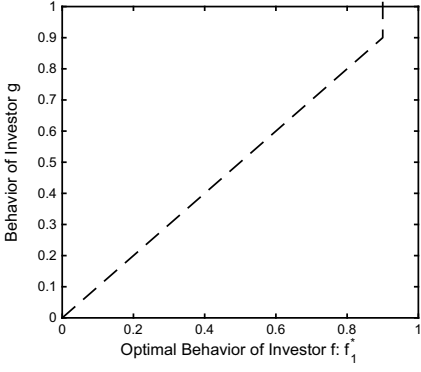
4 A numerical example

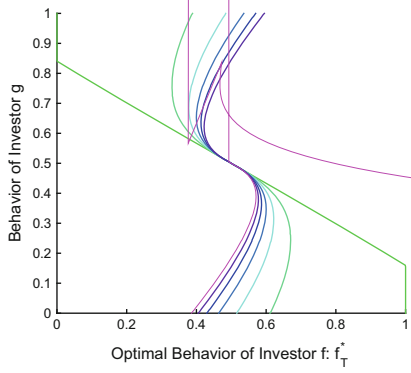
We construct a numerical example in this section to illustrate the results of Sects. 2 and 3. Consider the following two simple assets:

$$X_a = \begin{cases} \alpha & \text{with probability } p, \\ \beta & \text{with probability } 1 - p, \end{cases} \quad X_b = \gamma \quad \text{with probability } 1.$$

In this case, asset a is risky and asset b is riskless. The expected relative wealth of investor f in period T is explicitly given by :

$$\mathbb{E} \left[q_T^f \right] = \sum_{k=0}^T \binom{T}{k} p^k (1-p)^{T-k} \left(\frac{\alpha}{\beta} \right)^k \left(\frac{\gamma}{\beta} \right)^{T-k}$$





rather than on the absolute wealth. Relative wealth is important financially because success and satisfaction are sometimes measured by investors relative to the success of others (Robson 1992; Frank 1990, 2011; Bakshi and Chen 1996; Clark and Oswald 1996; Corneo and Jeanne 1997; Clark et al. 2008). Our model considers the case of two investors in a non-game-theoretic framework. We show how the optimal behavior of one investor is dependent on the other investor's behavior, which might be far from the Kell criterion. While some of our results are already known in the finance literature or the population genetics literature, they are not known together in both, and therefore they are included for completeness.

We consider myopic investors who maximize their expected relative wealth over a single period, and investors who maximize their relative wealth over multiple periods. Similar consequences hold in both cases. When one investor is wealthier than the other, that investor should roughly mimic the other's behavior in being more or less aggressive than the Kell criterion. Conversely, when one investor is poorer than the other, that investor should roughly act in the opposite manner of the other investor (Orr 2017).

As described above, it should be possible to design empirical biological studies to test the ideas of this paper. For example, one could design an experimental evolutionary study with a riskless condition (with constant fitness, corresponding to a fixed payoff) and a risky condition (with variable fitness, corresponding to different payoffs), much like the numerical example considered in Sect. 4. More generally, one could design an experimental environment with two random fitnesses that follow two different distributions. By varying the proportion of each population type exposed to each environment, one could create any type of investor as described in our model. Eventually, one would observe the growth of different types of investors to test various predictions about relative wealth in this paper.

Acknowledgment Research support from the MIT Laboratory for Financial Engineering and the University of Rochester is gratefully acknowledged.

A Appendix A: Proof

Proof of Proposition 1 See Brennan and Lo (2011). □

Proof of Proposition 2 The first partial derivative of $\mathbb{E}[q_1^f]$ to f is:

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f} = \lambda(1 - \lambda) \mathbb{E} \left[\frac{(X_a - X_b)\omega^g}{(\lambda\omega^f + (1 - \lambda)\omega^g)^2} \right].$$

The second partial derivative of $\mathbb{E}[q_1^f]$ to f is:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f^2} = -2\lambda^2(1 - \lambda) \mathbb{E} \left[\frac{(X_a - X_b)^2 \omega^g}{(\lambda\omega^f + (1 - \lambda)\omega^g)^3} \right] \leq 0,$$

which indicates that $\mathbb{E}[q_1^f]$ is a concave function of f . Therefore, it suffices to consider the value of the first partial derivative at its endpoints 0 and 1.

$$f_1^* = \begin{cases} 1 & \text{if } \frac{\partial \mathbb{E}[q_1^f]}{\partial f} \Big|_{f=1} > 0, \\ 0 & \text{if } \frac{\partial \mathbb{E}[q_1^f]}{\partial f} \Big|_{f=0} < 0, \\ \text{solution to } \frac{\partial \mathbb{E}[q_1^f]}{\partial f} = 0 & \text{otherwise.} \end{cases}$$

Proposition 2 follows from trivial simplifications of the above equation. □

Proof of Proposition 3 Consider $\frac{\partial \mathbb{E}[q_1^f]}{\partial f}$ when $f = g$:

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f} \Big|_{f=g} = \lambda(1 - \lambda) \mathbb{E} \left[\frac{X_a - X_b}{fX_a + (1 - f)X_b} \right].$$

Note that the righthand side consists of a factor that also appears in the first order condition (4) of the Kell criterion. Therefore its sign is determined by whether f is larger than f^{Kelly} :

$$\frac{\partial \mathbb{E}[q_1^f]}{\partial f} \Big|_{f=g} \begin{cases} > 0 \text{ if } f = g < f^{Kelly}, \\ = 0 \text{ if } f = g = f^{Kelly}, \\ < 0 \text{ if } f = g > f^{Kelly}. \end{cases} \tag{A.1}$$

Since $\mathbb{E}[q_1^f]$ is concave as a function of f for an g , we know that:

$$f_1^* \begin{cases} > g \text{ if } g < f^{Kelly}, \\ = g \text{ if } g = f^{Kelly}, \\ < g \text{ if } g > f^{Kelly}, \end{cases}$$

which completes the proof. □

Proof of Proposition 4 The cross partial derivative of $\mathbb{E}[q_1^f]$ is:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g} = \lambda(1 - \lambda) \mathbb{E} \left[\frac{(X_a - X_b)^2 (\lambda \omega^f - (1 - \lambda) \omega^g)}{(\lambda \omega^f + (1 - \lambda) \omega^g)^3} \right].$$

Consider $\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g}$ when $f = g = f^{Kelly}$:

$$\frac{\partial^2 \mathbb{E}[q_1^f]}{\partial f \partial g} \Big|_{f=g} = 2\lambda(1 - \lambda) \left(\lambda - \frac{1}{2} \right) \mathbb{E} \left[\left(\frac{X_a - X_b}{fX_a + (1 - f)X_b} \right)^2 \right] \begin{cases} < 0 \text{ if } \lambda < \frac{1}{2}, \\ = 0 \text{ if } \lambda = \frac{1}{2}, \\ > 0 \text{ if } \lambda > \frac{1}{2}. \end{cases}$$

The first order condition (A.1) is 0 when $f = g = f^{Kelly}$, so when g is near f^{Kelly} , the sign of the first order condition is determined by whether λ is greater than, equal to, or

less than $1/2$. For example, if $\lambda < 1/2$, then the derivative of the first order condition (A.1) with respect to g is negative, which implies that the first order condition is negative when $g = f^{Kelly} + \epsilon$, where ϵ is a small positive quantity. Therefore, when $g = f^{Kelly} + \epsilon$, f_1^* is smaller than f^{Kelly} . The cases when $\lambda > 1/2$ and $\lambda = 1/2$ follow similarly. \square

Proof of Proposition 5 The first partial derivative of $\mathbb{E}[q_T^f]$ to f is:

$$\frac{\partial \mathbb{E}[q_T^f]}{\partial f} = \frac{1 - \lambda}{\lambda} \mathbb{E} \left[\frac{\exp(T R_T^f) \sum_{i=1}^T \frac{X_{at} - X_{bt}}{f X_{at} + (1-f) X_{bt}}}{\lambda} \right]$$

- Blume, L., & Easley, D. (2006). If you're so smart, why aren't you rich? Belief selection in complete and incomplete markets. *Econometrica*, *74*(4), 929–966.
- Brennan, T. J., & Lo, A. W. (2011). The origin of behavior. *Quarterly Journal of Finance*, *1*, 55–108.
- Browne, S. (1999). Reaching goals by a deadline: Digital options and continuous-time active portfolio management. *Advances in Applied Probability*, *31*(2), 551–577.
- Browne, S., & Whitt, W. (1996). Portfolio choice and the Bayesian Kell criterion. *Advances in Applied Probability*, *28*, 1145–1176.
- Burnham, T. C., Dunlap, A., & Stephens, D. W. (2015). Experimental evolution and economics. *SAGE Open*. doi:10.1177/2158244015612524.
- Bushee, B. J. (1998). The influence of institutional investors on myopic R&D investment behavior. *Accounting Review*, *73*, 305–333.
- Clark, A. E., Frijters, P., & Shields, M. A. (2008). Relative income, happiness, and utility: An explanation for the Easterlin paradox and other puzzles. *Journal of Economic Literature*, *46*, 95–144.
- Clark, A. E., & Oswald, A. J. (1996). Satisfaction and comparison income. *Journal of Public Economics*, *61*(3), 359–381.
- Cooper, W. S., & Kaplan, R. H. (1982). Adaptive, coin-flipping: A decision-theoretic examination of natural selection for random individual variation. *Journal of Theoretical Biology*, *94*(1), 135–151.
- Corneo, G., & Jeanne, O. (1997). On relative wealth effects and the optimality of growth. *Economics Letters*, *54*(1), 87–92.
- De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1990). Noise trader risk in financial markets. *Journal of Political Economy*, *98*(4), 703–738.
- De Long, J. B., Shleifer, A., Summers, L. H., & Waldmann, R. J. (1991). The Survival of Noise traders in financial markets. *Journal of Business*, *64*(1), 1–19.
- Dunlap, A. S., & Stephens, D. W. (2014). Experimental evolution of prepared learning. *Proceedings of the National Academy of Sciences of USA*, *111*(32), 11750–11755.
- Evstigneev, I. V., Hens, T., & Schenk-Hopp, K. R. (2002). Market selection of financial trading strategies: Global stability. *Mathematical Finance*, *12*(4), 329–339.
- Evstigneev, I. V., Hens, T., & Schenk-Hopp, K. R. (2006). Evolutionarily stable stock markets. *Economic Theory*, *27*(2), 449–468.
- Felsenstein, J. (1976). The theoretical population genetics of variable selection and migration. *Annual Review of Genetics*, *10*(1), 253–280.
- Frank, R. H. (1985). *Choosing the right pond: Human behavior and the quest for status*. New York: Oxford University Press.
- Frank, S. A. (1990). When to copy or avoid an opponent's strategy. *Journal of Theoretical Biology*, *145*(1), 41–46.
- Frank, S. A. (2011). Natural selection. I. Variable environments and uncertain returns on investment. *Journal of Evolutionary Biology*, *24*, 2299–2309.
- Frank, S. A., & Slatkin, M. (1990). Evolution in a variable environment. *American Naturalist*, *136*, 244–260.
- Gillespie, J. H. (1973). Natural selection with varying selection coefficients: A haploid model. *Genetical Research*, *21*(02), 115–120.
- Gillespie, J. H. (1977). Natural selection for variances in offspring numbers: A new evolutionary principle. *The American Naturalist*, *111*(981), 1010–1014.
- Gillespie, J. H. (1991). *The causes of molecular evolution*. New York: Oxford University Press.
- Hakansson, N. H. (1970). Optimal investment and consumption strategies under risk for a class of utility functions. *Econometrica: Journal of the Econometric Society*, *38*, 587–607.
- Hansson, I., & Stuart, C. (1990). Malthusian selection of preferences. *The American Economic Review*, *80*, 529–544.
- Hens, T., & Schenk-Hopp, K. R. (2005). Evolutionarily stability of portfolio rules in incomplete markets. *Journal of Mathematical Economics*, *41*(1), 43–66.
- Hirshleifer, D., & Teoh, S. H. (2009). Thought and behavior contagion in capital markets. In *Handbook of financial markets: Dynamics and evolution. Handbooks in finance* (pp. 1–46). North Holland: Elsevier.
- Hirshleifer, J. (1977). Economics from a biological viewpoint. *Journal of Law and Economics*, *20*, 1–52.
- Hirshleifer, J. (1978). Natural economy versus political economy. *Journal of Social and Biological Structures*, *1*(4), 319–337.
- Hirshleifer, D., & Luo, G. Y. (2001). On the survival of overconfident traders in a competitive securities market. *Journal of Financial Markets*, *4*(1), 73–84.

- Hirshleifer, D., Subrahmanian, A., & Titman, S. (2006). Feedback and the success of irrational investors. *Journal of Financial Economics*, 81(2), 311–338.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2), 263–291.
- Kellard, J. L. (1956). A new interpretation of information rate. *IRE Transactions on Information Theory*, 2(3), 185–189.
- Kogan, L., Ross, S. A., Wang, J., & Westerfield, M. M. (2006a). The price impact and survival of irrational traders. *Journal of Finance*, 61(1), 195–229.
- Kogan, L., Ross, S. A., Wang, J., & Westerfield, M. M. (2006b). The price impact and survival of irrational traders. *Journal of Finance*, 61(1), 195–229.
- Lintner, J. (1965a). Security prices, risk, and maximal gains from diversification*. *The Journal of Finance*, 20(4), 587–615.
- Lintner, J. (1965b). The valuation of risk assets and the selection of risk investments in stock portfolios and capital budgets. *The Review of Economics and Statistics*, 47, 13–37.
- Lo, A. W. (2004). The adaptive markets hypothesis. *Journal of Portfolio Management*, 30(5), 15–29.
- Lo, A. W. (2017). *Adaptive markets: Financial evolution at the speed of thought*. Princeton, NJ: Princeton University Press.
- Luo, G. Y. (1995). Evolution and market competition. *Journal of Economic Theory*, 67(1), 223–250.
- Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77–91.
- Martindale Smith, J. (1982). *Evolution and the theory of games*. Cambridge: Cambridge University Press.
- Merfeld, F., & Kawecki, T. J. (2002). Experimental evolution of learning ability in fruit flies. *Proceedings of the National Academy of Sciences of USA*, 99(22), 14274–14279.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society*, 34, 768–783.
- Orr, H. A. (2017). Evolution, finance, and the population genetics of relative wealth. *Journal of Bioeconomics, Special Issue on Experimental Evolution*. doi:10.1007/s10818-017-9254-1.
- Reilly, F., & Brown, K. (2011). *Investment analysis and portfolio management*. Boston: Cengage Learning.
- Robson, A. J. (1992). Status, the distribution of wealth, private and social attitudes to risk. *Econometrica: Journal of the Econometric Society*, 60, 837–857.
- Robson, A. J. (1996). A biological basis for expected and non-expected utility. *Journal of Economic Theory*, 68(2), 397–424.
- Robson, A. J. (2001a). The biological basis of economic behavior. *Journal of Economic Literature*, 39(1), 11–33.
- Robson, A. J. (2001b). Would nature give individuals utility functions? *Journal of Political Economy*, 109(4), 900–914.
- Robson, A. J., & Samuelson, L. (2009). The evolution of time preference with aggregate uncertainty. *American Economic Review*, 99(5), 1925–1953.
- Rogers, A. R. (1994). Evolution of time preference by natural selection. *American Economic Review*, 84(3), 460–481.
- Samuelson, L. (2001). Introduction to the evolution of preferences. *Journal of Economic Theory*, 97(2), 225–230.
- Sandroni, A. (2000). Do markets favor agents able to make accurate predictions? *Econometrica*, 68(6), 1303–1341.
- Sandroni, A. (2005). Market selection when markets are incomplete. *Journal of Mathematical Economics*, 41(1), 91–104.
- Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19, 425–442.
- Strotz, R. H. (1955). Myopic and inconsistent dynamic utility maximization. *The Review of Economic Studies*, 23, 165–180.
- Strotz, K. (1983). Myopic utility functions on sequential economies. *Journal of Mathematical Economics*, 11(3), 267–276.
- Thaler, R. H., Tversky, A., Kahneman, D., & Schwartz, A. (1997). The effect of myopia and loss aversion on risk taking: An experimental test. *The Quarterly Journal of Economics*, 112, 647–661.
- Thorp, E. O. (1971). Portfolio choice and the Kelly criterion. In *Proceedings of the Business and Economics Section of the American Statistical Association* (pp. 215–224).
- Trenor, J. L. (1965). How to rate management of investment funds. *Harvard Business Review*, 43(1), 63–75.

- Waldman, M. (1994). Systematic errors and the theory of natural selection. *American Economic Review*, 84(3), 482–497.
- Yan, H. (2008). Natural selection in financial markets: Does it work? *Management Science*, 54(11), 1935–1950.
- Zhang, R., Brennan, T. J., & Lo, A. W. (2014a). Group selection as behavioral adaptation to systematic risk. *PLoS ONE*, 9(10), e110848.
- Zhang, R., Brennan, T. J., & Lo, A. W. (2014b). The origin of risk aversion. *Proceedings of the National Academy of Sciences of USA*, 111(50), 17777–17782.