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Gro p Selec ion as Beha ioral Adap a ion o S s ema ic Risk

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Abstract

Despite many compelling applications in economics, sociobiology, and evolutionary psychology, group selection is still one of the most hotly contested ideas in evolutionary biology. Here we propose a simple evolutionary model of behavior and show that what appears to be group selection may, in fact, simply be the consequence of natural selection occurring in stochastic environments with reproductive risks that are correlated across individuals. Those individuals with highly correlated risks will appear to form "groups", even if their actions are, in fact, totally autonomous, mindless, and, prior to selection, uniformly randomly distributed in the population. This framework implies that a separate theory of group selection is not strictly necessary to explain observed phenomena such as altruism and cooperation. At the same time, it shows that the notion of group selection does captures a unique aspect of evolution—selection with correlated reproductive risk–that may be sufficiently widespread to warrant a separate term for the phenomenon.

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Introduction

Since the publication of the path-breaking work of Wynne-Edwards [1] and Hamilton [2,3] in the 1960s, the theory of evolution has been applied to much broader contexts than selfreplicating genes. By appealing to notions of inclusive fitness and kin and group selection, compelling explanations for previously inexplicable behaviors such as altruism and cooperation have been developed. This approach has generated a number of additional insights such as reciprocity [4], the Price equation [5], sociobiology [6,7], and the theory of multi-level selection [8]. Moreover, empirical studies have lent further support to the theory of group and kin selection, including social behavior in bacteria [9–11], sterility in social insects [3,12], and the avoidance of cannibalism in salamanders [13].

The behavioral implications of group selection have also received considerable attention from economists, who have used evolutionary models to explain apparent conflicts between individual rationality and human behavior [14,15], including attitudes toward risk and utility functions [16,17], time preference [18], and financial markets [19,20]. As an alternative to the

traditional view that "market prices fully reflect all available information" [21,22], the Adaptive Markets Hypothesis [23] provides an evolutionary interpretation of financial market dynamics.

Despite these applications, group selection is still one of the most hotly contested issues in evolutionary biology. An enormous body of research has been dedicated to understanding the relationship between genetic, organismic, and group selection [24–26], and the relationship between group and kin selection [27–32]. Critics over the last few decades have argued forcefully against group selection as a major mechanism of evolution, and recent attempts to revive it [30,33,34] have been met with swift and broad rebuke [35,36].

Here we propose a reconciliation of the two opposing perspectives by arguing that what appears to be group selection may, in fact, simply be the consequence of natural selection occurring in stochastic environments with reproductive risks that are correlated across individuals. Those individuals with highly correlated risks will appear to form "groups", even if their actions are, in fact, totally autonomous, mindless, and, prior to natural selection, uniformly randomly distributed in the population.

We illustrate our approach with the following simple example. Consider a population of individuals, each facing a binary choice between one of two possible actions, a and b, and suppose the environment consists of two possible states of nature, rain or sunshine, with probability 20% and 80%, respectively. If it rains, action a leads to 0 offspring for any given individual and action bleads to 3 offspring; if it shines, the reverse occurs and action aleads to 3 offspring while action b leads to 0 offspring. From an individual's perspective, choosing a will lead to more reproductive success on average given the higher likelihood of sunshine. However, if all individuals in the population behaved in this manner, and rain or sunshine occurred for all individuals at the same time, the first time that a negative environment appears, the entire population of individuals that always choose a will become extinct. If we assume that offspring behave identically to their parents (perfect transmission of traits across generations), the behavior "always choose a" cannot survive over time. In fact, we show below that the behavior with the highest reproductive success over time in this very specialized example is to randomize between a and b using the same probability as the probability of sunshine, 80%; the group of individuals exhibiting this probability-matching behavior achieves the maximum possible growth rate. As a result, it appears as though selection operates at the group level and that this group—all individuals i who randomize their actions with probability $p_i = 80\%$ —is the fittest.

The key to this outcome is the fact that the reproductive risk facing all individuals in the population, rain or sunshine, is perfectly correlated, which we refer to as systematic risk. If we had assumed, instead, that reproductive risk was idiosyncratic—that the state of nature is independently and identically distributed (IID) for each individual—then the evolutionarily dominant strategy is, in fact, the purely "selfish" one in which a is chosen all the time.

This framework demonstrates that a separate theory of group selection is not strictly necessary to explain observed phenomena such as altruism and cooperation. But our results also show that the notion of group selection does capture a unique aspect of evolution-selection with correlated reproductive risk-that may be sufficiently widespread and distinct to warrant a separate term for the phenomenon. There is no controversy around the fact that selection occurs at the genetic level because of the basic biology of reproduction. However, we show that selection can also appear to operate at coarser levels if environmental forces affect a particular subset of individuals in similar fashion, i.e., if their reproductive risks are highly correlated. We use the term "appear" intentionally because in our framework, selection does not occur at the level of the group, but the behavior that is evolutionarily dominant is consistent with some of the empirical implications of group selection.

By studying the impact of selection on behavior rather than on genes, we are able to derive evolutionary implications that cut across species, physiology, and genetic origins. In the same way that different magnifications of a microscope reveal different details of a specimen, applying evolutionary principles to behavioral variations leads to different insights that may be more relevant for economics, psychology, and behavioral ecology. Because evolution is essentially a passive "process of elimination" [37], selection and adaptation operate at all levels, from genes to populations, depending on the nature of the corresponding environmental challenges. Our focus on behavior as the object of selection is a different lens through which the effects of evolution may be studied.

In the remainder of this paper, we provide a review of the literature, followed by an introduction to the binary choice model with two systematic environmental factors. In this framework, we derive a behavioral adaptation to stochastic environments with systematic risk in which groups seem to be the unit of selection, but which is purely the result of natural selection operating on individuals in the population. We conclude with a brief discussion of the results. A generalization of the binary choice model to the multinomial case with multiple factors is provided in the Supplementary Information, as well as all proofs and derivations.

Literature Review

The literature on evolution and behavior can be overwhelming, spanning the disciplines of evolutionary biology, ecology, evolutionary and social psychology, and economics. While a comprehensive survey is well beyond the scope of this paper, we attempt to provide a representative sampling of the many strands that are most relevant to our focus.

The role of stochastic environments in evolution has been investigated extensively by biologists and ecologists. Stochastic environments cause high genetic variation and, in the extreme cases, extinction [38-44]. Environmental uncertainty that is associated with stochasticity over time [45-47] or heterogeneity across space [48] can cause natural selection to favor a gene that randomizes its phenotypic expression. Gillespie and Guess [49] describe a heuristic method to study selection in random environments. Frank [50-52] analyzes how variability in reproductive success affects fitness and relates it to the geometric mean principle. Geometric-mean fitness has also appeared in the financial context as the "Kelly criterion" for maximizing the geometric growth rate of a portfolio [53-56]. However, the motivation for geometric-mean fitness in population biology is considerably more compelling than in financial investments because maximizing the geometric-mean return of a portfolio is optimal only for individuals with a very specific risk preference, i.e., those with logarithmic utility functions [54].

In the evolutionary biology literature, Maynard Smith [57] has developed the concept of an "evolutionarily stable strategy", specific behaviors that survive over time by conferring reproductive advantages or "fitness", typically measured by the rate of population growth. Using this notion of fitness, Fretwell [58], Cooper and Kaplan [59], and Frank and Slatkin [60] observe that randomizing behavior can be advantageous in the face of stochastic environmental conditions. The impact of variability in reproductive success among individuals in a population has been shown to yield a kind of risk aversion (which increases average reproductive success) and "bet-hedging" (which reduces the variance of reproductive success) [47,61–67]. Frank and Slatkin [60] propose a framework that highlights the importance of correlations among individual reproductive success in determining the path of evolution.

Similar results have been derived in the behavioral ecology literature, in which the maximization of fitness via dynamic programming has been shown to yield several observed behaviors, including risk-sensitive foraging in non-human animal species [68– 79] and seed dispersal strategies in plants [80,81]. Recently, the neural basis of risk aversion has also received much attention as researchers discovered that the activity of specific neural substrates correlates with risk-taking and risk-averse behaviors [82–85].

The relationship between risk-spreading behavior and kin selection has also been considered. Yoshimura and Clark [86] show that the risk-spreading polymorphism, in which a given genotype consists of a mixture of two or more forms each employing different behavioral strategies [48,59], makes sense only for groups. Yoshimura and Jansen [87] argue that risk-spreading adaptation is a form of kin selection [59], and the strategies of kin can be very important in stochastic environments even if there are no interactions at all. Mcnamara [88] introduces the profile of a strategy and relates the geometric mean fitness to a deterministic game.

In the economics literature, evolutionary principles were first introduced to understand cooperation and altruism [89–91], and evolutionary game theory is considered a foundation for altruistic behavior [17,92]. Evolutionary models of behavior are especially important for economists in resolving conflicts between individual rationality and human behavior [14,18], including attitudes toward risk and utility functions [16,17,93–95], time preference [96–100], financial markets and firm selection [19,20,101,102], and the economic analysis of social institutions [103–105].

Evolutionary models of behavior have also been used to justify the existence of utility functions and to derive implications for their functional form [16,95,106] (see Robson [18] and Robson and Samuelson [99] for comprehensive reviews of this literature). For example, Robson [16] investigates expected and non-expected utility behaviors, and finds that randomized behavior may be optimal from a population perspective even though it is suboptimal from an individual perspective [107,108]. Robson [95] argues that the kind of predictable behavior capable of being captured by a utility function emerged naturally as an adaptive mechanism for individuals faced with repeated choices in a nonstationary environment. Robson and Samuelson [97] find that exponential discounting in utility functions is consistent with evolutionarily optimal growth of a population.

Binary Choice Model with Systematic Risk

Consider a population of individuals that live for one period, produce a random number of offspring asexually, and then die. During their lives, individuals make only one decision: they choose from two actions, a and b, and this results in one of two corresponding random numbers of offspring, x_a and x_b . Now suppose that each individual chooses action a with some probability $p \in [0,1]$ and action b with probability 1-p, denoted by the Bernoulli variable I^p , hence the number of offspring of an individual is given by the random variable:

$$x^p = I^p x_a + (1 - I^p) x_b,$$

where

$$I^{p} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p. \end{cases}$$

We shall henceforth refer to p as the individual's *behavior* since it completely determines how the individual chooses between action a and b. Note that p can be 0 or 1, hence we are not requiring individuals to randomize—this will be derived as a consequence of natural selection under certain conditions.

Now suppose that there are two independent environmental factors, λ_1 and λ_2 , that determine reproductive success, and that x_a and x_b are both linear combinations of these two factors:

$$\begin{cases} x_a = \beta_1 \lambda_1 + (1 - \beta_1) \lambda_2 \\ x_b = \beta_2 \lambda_1 + (1 - \beta_2) \lambda_2 \end{cases}$$

where $\lambda_i \ge 0$, $0 \le \beta_i \le 1$, i=1,2. Examples of such factors are weather conditions, the availability of food, or the number of predators in the environment. Because these factors affect the

fecundity of all individuals in the population, we refer to them as *systematic*, and we assume that:

average growth rate of individuals as a function of their type f in terms of both behavior p and characteristics (β_1 , β_2).

Over time, individuals with the largest growth rate will dominate the population geometrically fast [14]. The optimal factor loading α_1^* that maximizes (4) is given by:

$$\alpha_{1}^{*} = \begin{cases} 1 & \text{if } \mathbb{E}[\lambda_{1}/\lambda_{2}] > 1 \text{ and } \mathbb{E}[\lambda_{2}/\lambda_{1}] < 1 \\ \text{solution to (6)} & \text{if } \mathbb{E}[\lambda_{1}/\lambda_{2}] \ge 1 \text{ and } \mathbb{E}[\lambda_{2}/\lambda_{1}] \ge 1 \\ 0 & \text{if } \mathbb{E}[\lambda_{1}/\lambda_{2}] < 1 \text{ and } \mathbb{E}[\lambda_{2}/\lambda_{1}] > 1 \end{cases}$$
(5)

where α_1^* is defined implicitly in the second case of (5) by

$$\mathbb{E}\left[\frac{\lambda_1}{\alpha_1^*\lambda_1 + (1 - \alpha_1^*)\lambda_2}\right] = \mathbb{E}\left[\frac{\lambda_2}{\alpha_1^*\lambda_1 + (1 - \alpha_1^*)\lambda_2}\right].$$
 (6)

As a result, the growth-optimal type is $f^* = (p^*, \beta_1^*, \beta_2^*)$, which is given explicitly in Table 1.

The optimal characteristics and associated optimal behaviors in Table 1 show that, when α_1^* is 1 or 0, one of the factors, λ_1 or λ_2 , is significantly more important than the other, and the optimal strategy places all the weight on the more important factor. However, when α_1^* is strictly between 0 and 1, a combination of factors λ_1 and λ_2 is necessary to achieve the maximum growth rate. Individual characteristics (β_1^*, β_2^*) need to be distributed in such a way that one of the two choices of action puts more weight on one factor, while the other choice puts more weight on the other factor. Eventually, the behavior p^* randomizes between the two choices and therefore achieves the optimal combination of factors. This is a generalization of the "adaptive coin-flipping" strategies of Cooper and Kaplan [59], who interpret this behavior as a form of altruism because individuals seem to be acting in the interest of the population at the expense of their own fitness. However, Grafen [107] provides a different interpretation by proposing an alternate measure of fitness, one that reflects the growth rate of survivors.

This result may be viewed as a primitive form of herding behavior—where all individuals in the population choose to act in the same manner—especially if the relative environmental factors, $\mathbb{E}[\lambda_1/\lambda_2]$ and $\mathbb{E}[\lambda_2/\lambda_1]$, shift suddenly due to rapid environmental changes. To an outside observer, behaviors among individuals in this population may seem heterogenous before the shift, but will become increasingly similar after the shift, creating the appearance (but not the reality) of intentional coordination, communication, and synchronization. If the reproductive cycle is sufficiently short, this change in population-wide behavior may seem highly responsive to environmental changes, giving the impression that individuals are learning about their environment. This is indeed a form of learning, but it occurs at the population level, not at the individual level, and not within an individual's lifespan.

Individually Optimal versus Group Optimal Behavior

It is instructive to compare the optimal characteristics and behavior in Table 1 that maximize growth with the behavior that maximizes an individual's reproductive success. According to (1) and (2), the individually optimal behavior maximizes

$$\mathbb{E}\left[x^{p,\beta_1,\beta_2}\right] = \mathbb{E}\left[I^p x_a^{\beta_1} + (1-I^p) x_b^{\beta_2}\right]$$
$$= \alpha_1 \mathbb{E}[\lambda_1] + \alpha_2 \mathbb{E}[\lambda_2]$$

over p, β_1, β_2 . Therefore, the individually optimal factor loading, denoted by $\hat{\alpha}_1$, is simply:

$$\hat{\alpha}_{1} = \begin{cases} 1 & \text{if } \mathbb{E}[\lambda_{1}] > \mathbb{E}[\lambda_{2}] \\ 0 & \text{if } \mathbb{E}[\lambda_{1}] < \mathbb{E}[\lambda_{2}] \\ \text{arbitrary} & \text{if } \mathbb{E}[\lambda_{1}] = \mathbb{E}[\lambda_{2}]. \end{cases}$$

Given a particular environment (λ_1, λ_2) , the individually optimal behavior depends only on the expectation of two factors, and this selfish behavior is generally sub-optimal for the group.

In contrast, individuals of type $f^* = (p^*, \beta_1^*, \beta_2^*)$ described in Table 1 are optimal in the group sense, attaining the maximum growth rate as a group by behaving differently than the individually optimal behavior. We shall refer to f^* henceforth as the growth-optimal behavior to underscore the fact that it is optimal from the population perspective, not necessarily from the individual's perspective. This provides a prototype of group selection as a consequence of stochastic environments with systematic risk. We define groups to be individuals with the same characteristics. More precisely, in our model, individuals with the same (β_1, β_2) are considered a group. Nature selects the groups with optimal characteristics (β_1^*, β_2^*), and p^* is a reflection of different behaviors for each group.

Table 1. Opt	mal type $f^* = (p^*, \beta_1^*, \beta_2^*)$ for the binary choice model
-	

	Optimal characteristics	Optimal behavior
If $\alpha_1^* = 1$	$\{(\beta_1,\beta_2): \beta_1 = 1 \text{ or } \beta_2 = 1\}$	$p^* = \begin{cases} \frac{\alpha_1^* - \beta_2^*}{\beta_1^* - \beta_2^*} = 1 & \text{if } \beta_1^* = 1, \beta_2^* \neq 1 \\ \frac{\alpha_1^* - \beta_2^*}{\beta_1^* - \beta_2^*} = 0 & \text{if } \beta_1^* \neq 1, \beta_2^* = 1 \\ \text{arbitrary} & \text{if } \beta_1^* = \beta_2^* = 1 \end{cases}$
If $\alpha_1^* = 0$	$\{(\beta_1,\beta_2): \beta_1=0 \text{ or } \beta_2=0\}$	$p^{*} = \begin{cases} \frac{\alpha_{1}^{*} - \beta_{2}^{*}}{\beta_{1}^{*} - \beta_{2}^{*}} = 1 & \text{if } \beta_{1}^{*} = 0, \beta_{2}^{*} \neq 0\\ \frac{\alpha_{1}^{*} - \beta_{2}^{*}}{\beta_{1}^{*} - \beta_{2}^{*}} = 0 & \text{if } \beta_{1}^{*} \neq 0, \beta_{2}^{*} = 0\\ \text{arbitrary} & \text{if } \beta_{1}^{*} = \beta_{2}^{*} = 0 \end{cases}$
If $0 < \alpha_1^* < 1$	$\left\{ (\beta_1,\beta_2) : (\beta_1 - \alpha_1^*)(\beta_2 - \alpha_1^*) \le 0 \right\}$	$p^* = \begin{cases} \frac{\alpha_1^* - \beta_2^*}{\beta_1^* - \beta_2^*} & \text{if } \beta_1^* \neq \beta_2^* \\ \text{arbitrary} & \text{if } \beta_1^* = \beta_2^* \end{cases}.$

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Like altruism, cooperation, trust, and other behaviors that do not immediately benefit the individual, the growth-optimal characteristics and behaviors derived in our framework flourish because they allow these individuals to pass through the filter of natural selection. However, unlike theories of group selection that are based on sexual reproduction and genetic distance, our version of group selection is based on behavior itself. Those individuals with types other than f^* will not reproduce as quickly, hence from an evolutionary biologist's perspective, group selection is operating at the level of those individuals with characteristics (β_1^*, β_2^*) and behaving according to p^* . Of course, we cannot measure all forms of characteristics and behavior as readily as we can measure genetic make-up, but in the stark case of the binary choice model, it is clear that selection can and does occur according to groups defined by characteristics and behavior.

The Supplementary Information contains a generalization of the binary choice model to multinomial choices with multiple environmental factors. In general, it is possible that the optimal growth rate μ^* corresponds to multiple groups, and each group corresponds to multiple optimal behaviors. In terms of group selection, this fact means that several different groups—each defined by a specific combination of characteristics—could simultaneously be optimal from an evolutionary perspective. Within each group, natural selection will determine the behavior that achieves the optimal growth rate. The optimal behavior is not necessarily unique for each group. Also, the optimal behaviors for different groups might overlap.

A Numerical Example

Consider an island that is isolated from the rest of the world, and suppose λ_1 is a measure of weather conditions of the local environment, and λ_2 is a measure of the local environment's topography where, without loss of generality, we assume that larger values of each factor are more conducive to reproductive success. Moreover, λ_1 and λ_2 are independent random variables

implications of group selection. We purposefully model individuals in our population as mindless creatures engaging in random choices to demonstrate that group-like selection can arise purely through common factors in reproductive success. If we include

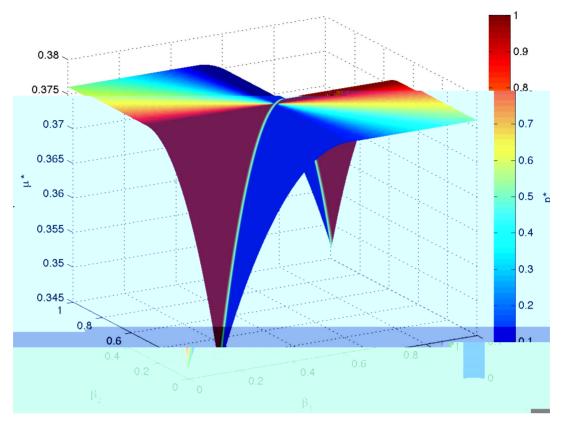


Figure 2. The optimal log-geometric-average growth rate for each group in the numerical example of group selection. doi:10.1371/journal.pone.0110848.g002

multiple groups achieve the same optimal growth rate but through different means, i.e., many combinations of behavior and characteristics can be optimal, leading to considerable variation in the types of behavior and characteristics in the population.

Hamilton's great insight was that individual fitness is not maximized by social evolution; inclusive fitness is [2, 3]. The idea that something other than the individual organism could be the fitness-maximizing unit was completely revolutionary at the time and opened new research areas that are still being explored [111]. We have shown that, in addition to Hamilton's insight, individuals with highly correlated risks will appear to form "groups" in evolution, even if their actions are, in fact, totally autonomous, mindless, and, prior to natural selection, uniformly randomly distributed in the population. Although this result seems to eliminate the need for a separate theory of group selection, the unique and important evolutionary implications of multiple sources of correlated systematic risk suggest that a separate term for this phenomenon may be worthwhile.

References

- Wynne-Edwards VC (1963) Intergroup selection in the evolution of social systems. Nature 200: 623–626.
- Hamilton WD (1963) The evolution of altruistic behavior. American Naturalist 97: 354–356.
- Hamilton WD (1964) The genetical evolution of social behavior. i and ii. Journal of Theoretical Biology 7: 1–52.
- Trivers RL (1971) The evolution of reciprocal altruism. The Quarterly Review of Biology 46: 35–57.
- 5. Price GR (1970) Selection and covariance. Nature 227: 520–521.
- Wilson EO (1975) Sociobiology: The New Synthesis. Cambridge, MA: Harvard University Press
- Wilson EO (2005) Kin selection as the key to altruism: its rise and fall. Social Research 72: 159166.

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Supporting Information

Text S1 The general multinomial choice model and proofs of all the results in the main text are provided in this document. (PDF)

Author Contributions

Conceived and designed the experiments: RZ TB AL. Performed the experiments: RZ TB AL. Analyzed the data: RZ TB AL. Contributed reagents/materials/analysis tools: RZ TB AL. Wrote the paper: RZ TB AL.

- Wilson DS, Sober E (1994) Reintroducing group selection to the human behavioral sciences. Behavioral and Brain Sciences 17: 585–607.
- Ben-Jacob E, Schochet O, Tenenbaum A, Cohen I, Czirok A, et al. (1994) Generic modelling of cooperative growth patterns in bacterial colonies. Nature 368: 46–49.
- Ben-Jacob E, Becker I, Shapira Y, Levine H (2004) Bacterial linguistic communication and social intelligence. TRENDS in Microbiology 12: 366– 372.
- Ingham C, Ben-Jacob E (2008) Swarming and complex pattern formation in paenibacillus vortex studied by imaging and tracking cells. BMC Microbiology 8: 36.
- Queller DC, Strassmann JE (1998) Kin selection and social insects. Bioscience 48: 165–175.

- Pfennig DW, Collins JP, Ziemba RE (1999) A test of alternative hypotheses for kin recognition in cannibalistic tiger salamanders. Behavioral Ecology 10: 436– 443.
- Brennan TJ, Lo AW (2011) The origin of behavior. Quarterly Journal of Finance 1: 55–108.
- Brennan TJ, Lo AW (2012) An evolutionary model of bounded rationality and intelligence. PloS one 7: e50310.
- Robson AJ (1996a) A biological basis for expected and non-expected utility. Journal of Economic Theory 68: 397–424.
- Samuelson L (2001) Introduction to the evolution of preferences. Journal of Economic Theory 97: 225–230.
- Robson AJ (2001a) The biological basis of economic behavior. Journal of Economic Literature 39: 11–33.
- Blume L, Easley D (1992) Evolution and market behavior. Journal of Economic Theory 58: 9–40.
- Kogan L, Ross SA, Wang J, Westerfield MM (2006) The price impact and survival of irrational traders. Journal of Finance 61: 195–229.
- Samuelson PA (1965) Proof that properly anticipated prices fluctuate randomly. Industrial Management Review 6: 41–49.
- Fama EF (1970) Efficient capital markets: A review of theory and empirical work. Journal of Finance 25: 383–417.
- Lo AW (2004) The adaptive markets hypothesis: Market efficiency from an evolutionary perspective. Journal of Portfolio Management 30: 15–29.
- Wilson DS (1975) A theory of group selection. Proceedings of the National Academy of Sciences 72: 143–146.
- 25. Wright S (1980) Genic and organismic selection. Evolution 34: 825-843.
- Williams GC (1996) Adaptation and natural selection: a critique of some current evolutionary thought. Princeton, NJ: Princeton University Press.
- Maynard Smith J (1964) Group selection and kin selection. Nature 201: 1145– 1147.
- Queller DC (1992) Quantitative genetics, inclusive fitness, and group selection. American Naturalist 139: 540–558.
- Wild G, Gardner A, West SA (2009) Adaptation and the evolution of parasite virulence in a connected world. Nature 459: 983–986.
- Nowak MA, Tarnita CE, Wilson EO (2010) The evolution of eusociality. Nature 466: 1057–1062.
- Wade MJ, Wilson DS, Goodnight C, Taylor D, Bar-Yam Y, et al. (2010) Multilevel and kin selection in a connected world. Nature 463: E8–E9.
- Frank SA (2013) Natural selection. vii. history and interpretation of kin selection theory. Journal of Evolutionary Biology 26: 1151–1184.
- Wilson DS, Wilson EO (2008) Evolution "for the good of the group". American Scientist 96: 380–389.
- 34. Wilson EO (2013) The Social Conquest of the Earth. New York: Liveright.
- Abbot P, Abe J, Alcock J, Alizon S, Alpedrinha JAC, et al. (2011) Inclusive fitness theory and eusociality. Nature 471: E1–E4.
- Pinker S (2012) The false allure of group selection. Technical report, Edge.com. Available: http://edge.org/conversation/the-false-allure-of-group-selection.
- 37. Mayr E (2001) What Evolution Is. New York: Basic Books.
- Lynch M, Lande R (1993) Evolution and extinction in response to environmental change. In: Karieva PM, Kingsolver JG, Huey RB, editors, Biotic Interactions and Global Change, Sunderland, MA: Sinauer Associates. pp. 235–250.
- Burger R, Lynch M (1995) Evolution and extinction in a changing environment: A quantitative-genetic analysis. Evolution 49: 151–163.
- Burger R, Gimelfarb A (2002) Fluctuating environments and the role of mutation in maintaining quantitative genetic variation. Genetical Research 80: 31–46.
- Pekalski A (1998) A model of population dynamics. Physica A: Statistical Mechanics and its Applications 252: 325–335.
- Pekalski A (1999) Mutations and changes of the environment in a model of biological evolution. Physica A: Statistical Mechanics and its Applications 265: 255–263.
- Pekalski A (2002) Evolution of population in changing conditions. Physica A: Statistical Mechanics and its Applications 314: 114–119.
- De Blasio FV (1999) Diversity and extinction in a lattice model of a population with fluctuating environment. Physical Review 60: 5912–5917.
- Cohen D (1966) Optimizing reproduction in a randomly varying environment. Journal of theoretical biology 12: 119–129.
- Oster GF, Wilson EO (1979) Caste and Ecology in the Social Insects. Princeton, NJ: Princeton University Press.
- Bergstrom TC (2014) On the evolution of hoarding, risk-taking, and wealth distribution in nonhuman and human populations. Proceedings of the National Academy of Sciences 111: 10860–10867.
- Levins R (1968) Evolution in Changing Environments: Some Theoretical Explorations.(MPB-2) (Vol. 2). Princeton, NJ: Princeton University Press.
- Gillespie JH, Guess HA (1978) The effects of environmental autocorrelations on the progress of selection in a random environment. American Naturalist 112: 897–909.
- Frank SA (2011a) Natural selection. i. variable environments and uncertain returns on investment. Journal of Evolutionary Biology 24: 2299–2309.
- Frank SA (2011b) Natural selection. ii. developmental variability and evolutionary rate. Journal of Evolutionary Biology 24: 2310–2320.
- Frank SA (2012a) Natural selection. iii. selection versus transmission and the levels of selection. Journal of Evolutionary Biology 25: 227–243.

- Kelly JL Jr (1956) A new interpretation of information rate. Bell System Technical Journal 35: 917–926.
- Samuelson PA (1971) The fallacy of maximizing the geometric mean in long sequences of investing or gambling. Proceedings of the National Academy of Sciences of the United States of America 68: 2493–2496.
- Thorp EO (1971) Portfolio choice and the kelly criterion. Proceedings of the Business and Economics Section of the American Statistical Association: 215– 224.
- Thorp EO (2006) The kelly criterion in blackjack, sports betting and the stock market. In: Zenios SA, Ziemba WT, editors, Handbook of Asset and Liability Management Oxford, UK: North-Holland. pp. 385–428.
- 57. Maynard Smith J (1982) Evolution and the Theory of Games. Cambridge university press.
- Fretwell SD (1972) Populations in a seasonal environment. 5. Princeton University Press.
- Cooper WS, Kaplan RH (1982) Adaptive "coin-flipping": A decision-theoretic examination of natural selection for random individual variation. Journal of Theoretical Biology 94: 135–151.
- Frank SA, Slatkin M (1990) Evolution in a variable environment. American Naturalist 136: 244–260.
- 61. Slatkin M (1974) Hedging one's evolutionary bets. Nature 250: 704-705.
- Caraco T (1980) On foraging time allocation in a stochastic environment. Ecology: 119–128.
- Real LA (1980) Fitness, uncertainty, and the role of diversification in evolution and behavior. American Naturalist: 623–638.
- Rubenstein DI (1982) Risk, uncertainty and evolutionary strategies. Current problems in sociobiology Cambridge University Press, Cambridge: 91–111.
- Seger J, Brockmann HJ (1987) What is bet-hedging? Oxford surveys in evolutionary biology 4: 182–211.
- Beaumont HJ, Gallie J, Kost C, Ferguson GC, Rainey PB (2009) Experimental evolution of bet hedging. Nature 462: 90–93.
- Childs DZ, Metcalf C, Rees M (2010) Evolutionary bet-hedging in the real world: empirical evidence and challenges revealed by plants. Proceedings of the Royal Society B: Biological Sciences: rspb20100707.
- Real L, Caraco T (1986) Risk and foraging in stochastic environments. Annual Review of Ecology and Systematics: 371–390.
- 69. Stephens DW, Krebs JR (1986) Foraging theory. Princeton University Press.
- Deneubourg JL, Aron S, Goss S, Pasteels JM (1987) Error, communication and learning in ant societies. European Journal of Operational Research 30: 168– 172.
- Harder LD, Real LA (1987) Why are bumble bees risk averse? Ecology 68: 1104–1108.
- Pasteels JM, Deneubourg JL, Goss S (1987) Self-organization mechanisms in ant societies. i: Trail recruitment to newly discovered food sources. Experientia Supplementum.
- Mangel M, Clark CW (1988) Dynamic modeling in behavioral ecology. Princeton University Press.
- 74. Hölldobler B, Wilson EO (1990) The Ants. Cambirdge, MA: Belknap Press.
- 75. Kirman A (1993) Ants, rationality, and recruitment. Quarterly Journal of

- Alexander RD (1974) The evolution of social behavior. Annual Review of Ecology and Systematics 5: 325–383.
- Hirshleifer J (1977) Economics from a biological viewpoint. Journal of Law and Economics 20: 1–52.
- Hirshleifer J (1978) Natural economy versus political economy. Journal of Social and Biological Structures 1: 319–337.
- Bergstrom TC (2002) Evolution of social behavior: Individual and group selection. Journal of Economic Perspectives 16: 67–88.
- Waldman M (1994) Systematic errors and the theory of natural selection. American Economic Review 84: 482–497.
- Robson AJ (1996b) The evolution of attitudes to risk: Lottery tickets and relative wealth. Games and Economic Behavior 14: 190–207.
- Robson AJ (2001b) Why would nature give individuals utility functions? Journal of Political Economy 109: 900–914.
- Rogers AR (1994) Evolution of time preference by natural selection. American Economic Review 84: 460–481.
- Robson AJ, Samuelson L (2007) The evolution of intertemporal preferences. American Economic Review 97: 496–500.
- Robson AJ, Szentes B (2008) Evolution of time preferences by natural selection: Comment. American Economic Review 98: 1178–1188.
- Robson AJ, Samuelson L (2009) The evolution of time preferences with aggregate uncertainty. American Economic Review 99: 1925–1953.
- Burnham TC (2013) Toward a neo-darwinian synthesis of neoclassical and behavioral economics. Journal of Economic Behavior and Organization 90: S113–S127.
- Luo GY (1995) Evolution and market competition. Journal of Economic Theory 67: 223–250.
- Luo GY (1998) Market efficiency and natural selection in a commodity futures market. Review of Financial Studies 11: 647–674.
- Herrmann-Pillath C (1991) A darwinian framework for the economic analysis of institutional change in history. Journal of Social and Biological Structures 14: 127–148.

- van den Bergh JC, Gowdy JM (2009) A group selection perspective on economic behavior, institutions and organizations. Journal of Economic Behavior and Organization 72: 1–20.
- Safarzyńska K, van den Bergh JC (2010) Evolving power and environmental policy: Explaining institutional change with group selection. Ecological Economics 69: 743–752.
- Hansson I, Stuart C (1990) Malthusian selection of preferences. The American Economic Review: 529–544.
- 107. Grafen A (1999) Formal darwinism, the individual-as-maximizing-agent analogy and bet-hedging. Proceedings of the Royal Society of London Series B: Biological Sciences 266: 799–803.
- Curry PA (2001) Decision making under uncertainty and the evolution of interdependent preferences. Journal of Economic Theory 98: 357–369.
- Boomsma JJ, Beekman M, Cornwallis CK, Griffin AS, Holman L, et al. (2011) Only full-sibling families evolved eusociality. Nature 471: E4–E5.
- Strassmann JE, Page RE Jr, Robinson GE, Seeley TD (2011) Kin selection and eusociality. Nature 471: E5–E6.
- 111. Ferriere R, Michod RE (2011) Inclusive fitness in evolution. Nature 471: E6–E8.
- 112. Herre EA, Wcisło WT (2011) In defence of inclusive fitness theory. Nature 471: E8–E9.
- Lehmann L, Kekker L, West S, Roze D (2007) Group selection and kin selection: Two concepts but one process. Proceedings of the National Academy of Sciences 104: 6736–6739.
- Marshall JAR (2011) Group selection and kin selection: Formally equivalent approaches. Trends in Ecology and Evolution 26: 325–332.
- Lion S, Jansen VAA, Day T (2011) Evolution in structured populations: Beyond the kin versus group debate. Trends in Ecology and Evolution 26: 193– 201.