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$k: \quad , S: k \quad , \quad S \quad \text{广群 (groupoid)} \quad :$

- $S \quad G,$
- $b, s : G \rightarrow S,$
- $S \times S \quad \circ : G \times_{sSb} G \rightarrow G.$

$k \quad T,$

$(S(T) = \text{Hom}(T, S), G(T) = \text{Hom}(T, S), s, b, \circ)$

$, \quad S(T) \quad , G(T) \quad .$

$: \quad S \times S \quad \varepsilon : S \rightarrow G$

“ -1 ” : $G \rightarrow G,$

S G 表示 $V \in \text{Qcoh}(S)$, G .
 G $u : s^*V \xrightarrow{\sim} b^*V$:

- ε^*u ,

- $u \circ : G \times_{s,b} G \rightarrow G$ $\text{pr}_1^*u \circ \text{pr}_2^*u$.

S T , V ,
 $(S(T), G(T), s, b, \circ)$:

$$t : T \rightarrow S \quad t^*V,$$

$$g : G \rightarrow S \quad g^*u : s(g)^*V \rightarrow b(g)^*V.$$

S G 表示 $V \in \text{Qcoh}(S)$, G .

gerbe

S , Sch/k ,
 (prestack). fpqc $\mathcal{G}_{S:G}$.
 Sch/k \mathcal{F} ,
 Sch/k $\mathcal{F} \rightarrow \text{Qcoh}/k$.
 $: k$ T , $\mathcal{F}(T) \rightarrow \text{Qcoh}(T)$.
 $\text{Rep}(\mathcal{G}_{S:G}) \simeq \text{Rep}(S : G)$.

gerbe

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 (prestack). fpqc $\mathcal{G}_{S:G}$.

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 $: k$ T , $\mathcal{F}(T) \rightarrow Qcoh(T)$.
 $, Rep(\mathcal{G}_{S:G}) \simeq Rep(S : G)$.

gerbe

$\mathcal{G}_{S:G}$ gerbe
 $(s, b) : G \rightarrow S \times S$ fpqc
 G
 G S

Deligne 1990

k , k -张量范畴 (tensor category)
 k - \mathcal{C} $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$:

① (\mathcal{C}, \otimes) $\mathbb{1}$.

② $X \in \mathcal{C}$, X^\vee $\delta : \mathbb{1} \rightarrow X^\vee \otimes X$

$\text{ev} : X \otimes X^\vee \rightarrow \mathbb{1}$:

$X \xrightarrow{\text{id} \otimes \delta} X \otimes X^\vee \xrightarrow{\text{ev} \otimes \text{id}} X$ 和 $X^\vee \xrightarrow{\delta \otimes \text{id}} X^\vee \otimes X \xrightarrow{\text{id} \otimes \text{ev}} X^\vee$

③ \mathcal{C}

④ $k \xrightarrow{\sim} \text{Hom}(\mathbb{1}, \mathbb{1})$

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- (1) $\mathcal{T}, \mathcal{T}', F: \mathcal{T} \rightarrow \mathcal{T}'$
张量函子 (\otimes -functor)

- $F(X) \otimes F(Y) \rightarrow F(X \otimes Y),$
- ,

- \mathcal{T} 纤维函子 (fiber functor)

$$F: \mathcal{T} \rightarrow \text{Qcoh}(S) \quad S \text{ } k\text{-}$$

- \mathcal{T} 淡中范畴 (tannakian category)

$$\omega: \mathcal{T} \rightarrow \text{Qcoh}(S) \quad S \neq \emptyset.$$

$$u: \mathcal{T} \rightarrow S \text{ } k\text{-}, \quad \omega: \mathcal{T} \rightarrow \text{Qcoh}(S)$$

$$u^* \omega: \mathcal{T} \rightarrow \text{Qcoh}(T)$$

$$\mathcal{T}$$

$$\omega: \mathcal{T} \rightarrow \text{Vect}(K), \quad K \text{ } k$$

. (Deligne 1990)

$\mathcal{T}: k \quad , \omega : \mathcal{T} \rightarrow \text{Qcoh}(S): \quad , \quad S/k \neq \emptyset.$

- $\underline{\text{Aut}}_k^\otimes(\omega) \quad S \times S \quad .$
- $\omega \quad \mathcal{T} \xrightarrow{\sim} \text{Rep}(S : \underline{\text{Aut}}_k^\otimes(\omega)).$
- $\quad , \quad G \quad S \neq \emptyset \quad ,$
 $S \times S \quad .$
 $\omega \quad \text{Rep}(S : G) \quad G \quad ,$
 $\omega \quad : G \xrightarrow{\sim} \underline{\text{Aut}}_k^\otimes(\omega).$

. (Barr-Back)

$$(L, R) \quad , \quad LR \rightarrow \text{id}, \text{id} \rightarrow RL, \\ F = LR, \quad LA \rightarrow LRLA = F(LA) \quad LA \quad F \quad .$$

$$L : \mathcal{A} \rightarrow \mathcal{B} \quad (f, g) : A \rightrightarrows A' \\ (Lf, Lg) \quad , \quad K = \ker(f, g) \quad LK = \ker(Lf, Lg). \\ L \quad \mathcal{A} \quad F \quad \mathcal{B} \quad .$$

$${}_A M_B \quad (A, B) \quad , \quad A \quad , \quad B \\ , \quad B \quad {}_B M_A^\vee \quad (B, A) \quad .$$

$$L : E \rightarrow E \otimes_A {}_A M_B \quad A \quad B \quad , \\ R : F \rightarrow F \otimes_B {}_B M_A^\vee, \quad \text{Barr-Back} \quad , \\ A \quad {}_B M_A^\vee \otimes_A {}_A M_B \quad B \quad .$$

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$$\begin{aligned}
 k: & \quad , B_1, B_2: k \quad . \omega_i : \mathcal{C} \rightarrow (B_i)_{\text{ptf}} \quad , \\
 & \quad (B_i)_{\text{ptf}} \quad B_i \quad . \\
 & : (B_1, B_2) \quad L_k(\omega_1, \omega_2) \quad : \\
 & \quad X \in \mathcal{C}, \quad \omega_1(X)^\vee \otimes \omega_2(X) \rightarrow L_k(\omega_1, \omega_2) \\
 & \quad X, Y \in \mathcal{C}, \quad . \\
 & : \quad . \\
 & \quad L_k(\omega_1, \omega_3) \rightarrow L_k(\omega_1, \omega_2) \otimes L_k(\omega_2, \omega_3). \\
 & \quad . \\
 & \quad L_k(\omega) = L_k(\omega, \omega), \quad . \\
 & \quad , \omega(X)^\vee \otimes \omega(X) \rightarrow L_k(\omega) \quad \omega(X) \rightarrow \omega(X) \otimes L_k(\omega) \\
 & L_k(\omega) \quad \omega(X) \quad .
 \end{aligned}$$

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k , \mathcal{A} k - $Abel$,

Hom .

k $B, \omega : \mathcal{A} \rightarrow (B)_{\text{ptf}}$, kB

()

() .

$$Y \in \langle X \rangle \quad \langle Y \rangle \subseteq \langle X \rangle.$$

$$B \quad , \quad L_k(\omega|\langle Y \rangle) \rightarrow L_k(\omega|\langle X \rangle) \quad ,$$

B

A

 $\langle X \rangle$, $L_k(\omega)$

$$L_k(\omega|\langle X \rangle)$$



A

, ω

,

$$L_k(\omega_1, \omega_2) \otimes L_k(\omega_1, \omega_2) \rightarrow L_k(\omega_1, \omega_2).$$

A

, $L_k(\omega_1, \omega_2)$

$$B \quad . \quad \text{Spec } L_k(\omega_1, \omega_2)$$

$$\underline{\text{Hom}}_S^{\otimes}(\omega_1, \omega_2),$$

$$S = \text{Spec } B.$$

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$$\Lambda_{\mathcal{T}}(T_1, T_2) = T_1(X)^\vee \otimes T_2(X) \quad (\text{coend}), \quad \text{Ind}(\mathcal{T})$$

$$\Lambda_k(T_1, T_2) = \Lambda_{\mathcal{T} \otimes \mathcal{T}}(T_1 \times \mathbb{1}, \mathbb{1} \times T_2).$$

$$T : \mathcal{T} \otimes \mathcal{T} \rightarrow \mathcal{T}, \quad T(\Lambda_k(\text{id}, \text{id})) = \Lambda_{\mathcal{T}}(\text{id}, \text{id}),$$

$$\omega(\Lambda_k(\text{id}, \text{id})) = L_k(\omega).$$

$$\text{ev} : X^\vee \otimes X \rightarrow \mathbb{1} \quad \Lambda_{\mathcal{T}}(\text{id}, \text{id}) \rightarrow \mathbb{1},$$

$$\mathbb{1} \rightarrow \Lambda_{\mathcal{T}}(\text{id}, \text{id})$$

$$\mathcal{T} \quad S = \text{Spec } B$$

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Grassmannian

$G_{\mathcal{O}}$, G_F Ind-
 $Gr_G = G_F/G_{\mathcal{O}}$. Ind-

G GL_n , Gr_G Gr_{GL_n} .
 GL_n , Gr_{GL_n}
 Grassmannian

Gr_G : Beauville-Laszlo
 X , $x \in X$, $X^* = X \setminus \{x\}$.

R , $X_R = X \otimes_k R$.
 $Gr_G(R) = (\mathcal{E}, \beta)$ $\mathcal{E}|_{X_R}$ G - (torsor), isom
 $\beta : \mathcal{E}|_{X_R^*} \rightarrow \mathcal{E}^{\circ}|_{X_R^*}$

$G_{\mathcal{O}}$

$ev : G_{\mathcal{O}} \rightarrow G$ $t = 0$.
 (B, T) Borel , $I = ev^{-1}B$ G_F Iwahori ,
 $G_{\mathcal{O}}$ Parahori . Cartan :
 $G_{\mathcal{O}} \backslash G_F / G_{\mathcal{O}} = W \backslash (X_{\bullet} \times W) / W = X_{\bullet}^+$.

$\lambda \in X_{\bullet}$, $\mathbb{G}_m \rightarrow T$ $\mathbb{G}_{m,F} \rightarrow T_F$.
 t^{λ} $t \in \mathbb{G}_{m,F}$.

$G_{\mathcal{O}}$ Gr_G , X_{\bullet}^+ .
 $\lambda \in X_{\bullet}^+$, $G_{\mathcal{O}} t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}}$ $G_{\mathcal{O}}$.

$Gr_G^{\lambda} = G_{\mathcal{O}} t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}}$,
 $Perv_{G_{\mathcal{O}}}(Gr_G, A)$ $IC_{Gr_G^{\lambda}}$.

$$Gr_G \times Gr_G \xleftarrow{p} G_F \times Gr_G \xrightarrow{q} G_F \times^{G_O} Gr_G = Gr_G \otimes Gr_G \xrightarrow{m} Gr_G.$$

$$\mathcal{F}, \mathcal{G} \in \text{Perv}_{G_O}(Gr_G, A),$$

$$Gr_G \otimes Gr_G$$

$$\mathcal{F} \boxtimes \mathcal{G} \quad p^*(\mathcal{F} \boxtimes \mathcal{G}) = q^*(\mathcal{F} \boxtimes \mathcal{G}).$$

$$\mathcal{F} \star \mathcal{G} = m_*(\mathcal{F} \boxtimes \mathcal{G}), \quad \mathcal{F} \quad \mathcal{G} \quad .$$

$$Gr_G \times Gr_G \xleftarrow{p} G_F \times Gr_G \xrightarrow{q} G_F \times^{G_O} Gr_G = Gr_G \otimes Gr_G \xrightarrow{m} Gr_G.$$

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$$\mathcal{F} \star \mathcal{G} = m_*(\mathcal{F} \boxtimes \mathcal{G}), \quad \mathcal{F} \quad \mathcal{G} \quad .$$

$$), \quad (\ominus^r \quad A),$$

$$\text{Perv}_{G_{\mathcal{O}}}(Gr_G, A)$$

$$\lambda \in X_{\bullet}^+, \quad IC_{\lambda} = IC_{\overline{Gr_G^{\lambda}}}.$$

$$\text{Hom}(IC_{\lambda}, IC_{\mu}[1]) = 0$$

$$\lambda, \mu \in X_{\bullet}^+.$$

$$:$$

- $\overline{Gr_G^{\lambda}} = \sqcup_{\nu \leq \lambda} Gr_G^{\nu}$, $\dim \overline{Gr_G^{\lambda}} = \dim Gr_G^{\lambda} = \langle 2\rho, \lambda \rangle$,
- $\mathcal{H}^i(IC_{\lambda}) = 0$ $i < \langle 2\rho, \lambda \rangle$.

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• : G_F Kac-Moody , G_O
Parahori , .

• : Gr_G^λ , t^λ .

• : $C_{\lambda, \alpha} \setminus \{t^{\lambda - \alpha^\vee}\} \subset Gr_G^\lambda$, C_{λ, α^\vee} ,
 α , $\lambda, \lambda - \alpha^\vee \in X_\bullet^+$,
 : SL_2 ,



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Bruhat , $Gr_G = \sqcup_{w \in \widetilde{W}/W} IwG_{\mathcal{O}}/G_{\mathcal{O}}$.

$w_{\lambda} \in Wt^{\lambda}W$, $Iw_{\lambda}G_{\mathcal{O}}/G_{\mathcal{O}}$, Gr_G^{λ} .

$IC_{\lambda} = IC_{Iw_{\lambda}G_{\mathcal{O}}/G_{\mathcal{O}}}$.

$Fl_G = G_F/I \rightarrow Gr_G$, G/B^{-} .

$Iw_{\lambda}G_{\mathcal{O}}/G_{\mathcal{O}}$, $Iw_{\lambda}w_{\circ}I/I$.

, $IC_{IwI/I}$, $\overline{IwI/I}$

Bott-Samelson , .



$$\mathcal{F}, \mathcal{G} \in \text{Perv}_{G_0}$$

$$U = [B, B] \quad B \quad , \lambda \in X_{\bullet},$$

$$S_{\lambda} = U_F t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}} \quad U_F \quad Gr_G \quad .$$

:

- $\overline{S_{\lambda}} = \sqcup_{\nu \leq \lambda} S_{\nu};$

- $2\rho^{\vee} \quad , \quad \mathbb{G}_m \rightarrow T \hookrightarrow G$
 $\mathbb{G}_m \quad Gr_G \quad n \quad .$

$$S_{\lambda} = \{ x \in Gr_G \mid \lim_{s \rightarrow 0} 2\rho^{\vee}(s) \cdot x = t^{\lambda} \} .$$

$$U_F^-$$

$$T_{\lambda} = \{ x \in Gr_G \mid \lim_{s \rightarrow \infty} 2\rho^{\vee}(s) \cdot x = t^{\lambda} \} .$$

()-Braden's hyperbolic localization theorem

$$\begin{array}{l}
 \mathbb{G}_m \\
 S \in \mathcal{D}(X) \\
 \rho^* S = L \boxtimes S,
 \end{array}
 \quad
 \begin{array}{l}
 X \\
 L \\
 \mathbb{G}_m
 \end{array}
 , \quad
 \begin{array}{l}
 \rho : \mathbb{G}_m \times X \rightarrow X, \\
 \\
 .
 \end{array}$$

(Braden 2003)

$$\begin{array}{l}
 X \\
 n \\
 X^+ = \{ x \in X \mid \lim_{s \rightarrow 0} s \cdot x \text{ exists} \}, \\
 \\
 F = X^{\mathbb{G}_m} \\
 \circ \\
 n \\
 X^- = \{ x \in X \mid \lim_{s \rightarrow \infty} s \cdot x \text{ exists} \}. \\
 \circ
 \end{array}$$

$$f^\pm : F \rightarrow X^\pm, g^\pm : X^\pm \rightarrow X, \pi^\pm : X^\pm \rightarrow F.$$

$$S \in \mathcal{D}(X) \quad \mathbb{G}_m^- \quad , \quad :$$

- $(f^\pm)^* S = (\pi^\pm)_* S, (f^\pm)! S = (\pi^\pm)! S,$
- $(f^-)^*(g^-)! S \rightarrow (f^+)^!(g^+)^* S.$

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$$\overline{Gr_G^\lambda} \quad ,$$

$$H_{T_\mu}^k(Gr_G, IC_\lambda) \xrightarrow{\sim} H_c^k(S_\mu, IC_\lambda).$$

$$k \neq \langle 2\rho, \mu \rangle \quad .$$

$$\lambda \in X_{\bullet}^+, \mu \in X_{\bullet}$$

$$\dim \overline{Gr_G^\lambda} \cap S_\mu = \langle \rho, \lambda + \mu \rangle.$$

$$\mathcal{F} \in \text{Perv}_{G_O}(Gr_G)$$

$$H_c^k(S_\mu, \mathcal{F}) = 0, k > \langle 2\rho, \mu \rangle.$$

$$H_{T_\mu}^k(Gr_G, \mathcal{F}) = 0, k < \langle 2\rho, \mu \rangle.$$



()

: $\mathcal{F} \in \text{Perv}_{G_0}(Gr_G)$

- $H_c^{\langle 2\rho, \mu \rangle}(S_\mu, \mathcal{F}) = H_c^{\langle 2\rho, \mu \rangle}(\overline{S}_\mu, \mathcal{F}),$
- $H_c^{\langle 2\rho, \nu \rangle}(\overline{S}_\mu, \mathcal{F}) = H_c^{\langle 2\rho, \nu \rangle}(\overline{S}_\nu, \mathcal{F}), \nu \leq \mu,$
- $H_c^{\langle 2\rho, \mu \rangle + \text{odd}}(\overline{S}_\mu, \mathcal{F}) = 0.$

$$T_\mu, \quad F_\mu = H_c^{\langle 2\rho, \mu \rangle}(S_\mu, -), \quad H(Gr_G, \mathcal{F}) = \bigoplus_{\mu \in X_\bullet} F_\mu(\mathcal{F}).$$

$$H_{T_\mu}^{\langle 2\rho, \mu \rangle}(Gr_G, \mathcal{F}) \quad H^{\langle 2\rho, \mu \rangle}(Gr_G, \mathcal{F}) \quad H^{\langle 2\rho, \mu \rangle}$$

$$Gr_{G,X}(R) = \left(\begin{array}{l} X \\ (x, \mathcal{E}, \beta) \end{array} \right), \quad \begin{array}{l} Gr_{G,X} \\ x \in X(R), \mathcal{E} \text{ } X_R \text{ } G\text{-} \\ \beta : \mathcal{E}|_{X_R \setminus \Gamma_x} \rightarrow \mathcal{E}^\circ|_{X_R \setminus \Gamma_x} \end{array} \text{ isom}$$

Gr_{G,X^2} , Beilinson-Drinfeld Grassmannian.

- $Gr_{G,X^2}|_{X^2 \setminus \Delta} = (Gr_{G,X} \times Gr_{G,X})|_{X^2 \setminus \Delta}$.
- $Gr_{G,X^2}|_{\Delta} \simeq Gr_{G,X}$.

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$$U = X^2 \setminus \Delta, \quad Gr_{G,X} \otimes Gr_{G,X} \xrightarrow{m} Gr_{G,X^2}.$$

$$X = \mathbb{A}^1, \quad Gr_{G,X} = Gr_G \times X, \\ \mathcal{F} \in \text{Perv}_{G_O}(Gr_G), \quad \mathcal{F}_X \in \text{Perv}_{G_O,X}(Gr_{G,X}).$$

$$U, \quad \Delta, \quad \mathcal{F}_X \star_X \mathcal{G}_X, \\ \mathcal{F} \star \mathcal{G}[2], \\ U, \quad \Delta, \quad \mathcal{F} \boxtimes \mathcal{G}[2].$$

$$Gr_{G,X^2}$$

$$\mathbb{G}_m, \quad Gr_{G,X} \otimes Gr_{G,X}, \quad (t^{\mu_1}, t^{\mu_2})_{X^2}, \\ m, \quad \Delta, \\ \mu \in X \bullet, \quad S_\mu(X^2).$$

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$$\begin{array}{ccc}
 S_\mu(X^2) & \Delta & \\
 U & & \\
 & & \text{F } S_\mu, \\
 & & S_{\mu_1} \times S_{\mu_2}. \\
 & & \mu_1 + \mu_2 = \mu \\
 & & \\
 \mathcal{F}_X \star_X \mathcal{G}_X & & \mathcal{H}_c^{\langle 2\rho, \mu \rangle - 2}(S_\mu(X^2)/X^2, \mathcal{F}_X \star_X \mathcal{G}_X). \\
 \Delta & & \\
 U & & H_c^{\langle 2\rho, \mu \rangle}(S_\mu, \mathcal{F} \star \mathcal{G}) = F_\mu(\mathcal{F} \star \mathcal{G}). \\
 H^{\langle 2\rho, \mu \rangle} & \text{F} & \\
 & S_{\mu_1} \times S_{\mu_2}, \mathcal{F} \boxtimes \mathcal{G} & = \text{L} \\
 & \mu_1 + \mu_2 = \mu & \mu_1 + \mu_2 = \mu \\
 & & F_{\mu_1}(\mathcal{F}) \otimes F_{\mu_2}(\mathcal{G}).
 \end{array}$$

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$$S_\mu(X^2) \quad \Delta \quad \mathbb{F} \quad S_\mu, \\ U \quad \quad \quad S_{\mu_1} \times S_{\mu_2}. \\ \mu_1 + \mu_2 = \mu$$

$$\mathcal{F}_X \star_X \mathcal{G}_X \quad \mathcal{H}_c^{\langle 2\rho, \mu \rangle - 2}(S_\mu(X^2)/X^2, \mathcal{F}_X \star_X \mathcal{G}_X).$$

$$\Delta \quad \mathcal{H}_c^{\langle 2\rho, \mu \rangle}(S_\mu, \mathcal{F} \star \mathcal{G}) = F_\mu(\mathcal{F} \star \mathcal{G}).$$

$$H^{\langle 2\rho, \mu \rangle} \quad \mathbb{F} \quad S_{\mu_1} \times S_{\mu_2}, \mathcal{F} \boxtimes \mathcal{G} = \quad \mathbb{L} \quad F_{\mu_1}(\mathcal{F}) \otimes F_{\mu_2}(\mathcal{G}). \\ \mu_1 + \mu_2 = \mu \quad \mu_1 + \mu_2 = \mu$$

- X^2
- $\mathcal{H}^k(Gr_{G, X^2}, \mathcal{F}_X \star_X \mathcal{G}_X)$

$$\text{Sat}_G = \text{Perv}_{G_0}(Gr_G).$$

$$F = H(Gr_G, -) \quad \text{Sat}_G$$

$$\text{SVect}(A),$$

$$\text{Vect}(A)$$

$$\text{Sat}_G \quad \text{Rep}(\mathcal{G}, A).$$

$$\text{Sat}_G$$

$$, \quad \mathcal{G}$$

$$\text{Sat}_G$$

$$X, \quad \langle X \rangle$$

$$G$$

$$\text{Sat}_G$$

$$\mathcal{G}$$

$$\mathcal{G} \quad \mathcal{G}$$

(\quad)

$$\begin{array}{l}
 T \quad \mathbb{G}_m^n, \\
 \text{Gr}_T \quad X_\bullet(T) \\
 \text{Sat}_T \quad X_\bullet(T) \\
 \mathcal{P} = \mathcal{P} \\
 F = \prod_{\mu \in X_\bullet} F_\mu \\
 \text{Sat}_G \rightarrow \text{Vect}_{X_\bullet}(A) \rightarrow \text{Vect}(A). \\
 \mathcal{G} \quad X_\bullet^+, \quad \mathcal{G}
 \end{array}
 \quad : \quad
 \mathcal{P} \rightarrow \mathcal{G}.$$

$()$

$$\begin{aligned}
 & 2\rho \in X^\bullet = \hat{X}_\bullet, \\
 & \mathcal{G} \quad , \quad \text{Borel} \quad , \quad \mathfrak{p}. \\
 & \lambda \in X_\bullet^+, \quad IC_\lambda \quad \lambda \quad F(IC_\lambda) \quad . \\
 & \quad , \quad \mathcal{G} \quad \lambda \quad , \quad X_\bullet^+. \\
 & X_\bullet^+ = \hat{\mathcal{G}}_\bullet^+. \\
 & \quad , F(IC_\lambda)_\mu \quad \mu \leq \lambda. \quad \hat{X}_\bullet \\
 & (\text{root lattice}) \hat{\mathcal{Q}}_+ \quad \mathcal{Q}_+^\vee. \\
 & \quad \mathcal{G} \quad .
 \end{aligned}$$

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• Zeta

Hodge

$$\begin{array}{l}
 k: \quad 0 \quad , \quad \bar{k}, \text{ Galois} \quad \Gamma = \text{Gal}(\bar{k}/k). \\
 X: k \quad . \quad \ell, H_\ell(X): X \quad \overset{\ell}{\mathbb{Q}} \quad , \\
 H_{\text{dR}}(X): X \quad \text{de Rham} \quad . \quad H_{\mathbb{A}}(X) = H_{\text{dR}}(X) \times_{\ell} H_\ell(X). \\
 \sigma : k \hookrightarrow \mathbb{C}, H_\sigma(X) \quad \sigma X = X \otimes_{k, \sigma} \mathbb{C} \quad \text{Betti} \quad . \\
 \quad : H_\sigma(X) \otimes (\mathbb{C} \times \mathbb{A}_f) \xrightarrow{\sim} H_{\mathbb{A}}(\sigma X). \\
 \sigma^* \quad H_\sigma(X) \hookrightarrow H_{\mathbb{A}}(\sigma X).
 \end{array}$$

Hodge

$$\begin{aligned}
 k: & \quad 0 \quad , \quad \bar{k}, \text{ Galois} \quad \Gamma = \text{Gal}(\bar{k}/k). \\
 X: & \quad k \quad . \quad \ell, H_\ell(X): X \xrightarrow{\ell} \mathbb{Q}^\ell, \\
 H_{\text{dR}}(X): & \quad X \quad \text{de Rham} \quad . \quad H_{\mathbb{A}}(X) = H_{\text{dR}}(X) \times_{\ell} H_\ell(X). \\
 \sigma: & \quad k \hookrightarrow \mathbb{C}, H_\sigma(X) \quad \sigma X = X \otimes_{k, \sigma} \mathbb{C} \quad \text{Betti} \quad . \\
 & \quad : H_\sigma(X) \otimes (\mathbb{C} \times \mathbb{A}_f) \xrightarrow{\sim} H_{\mathbb{A}}(\sigma X). \\
 \sigma^* & \quad H_\sigma(X) \hookrightarrow H_{\mathbb{A}}(\sigma X).
 \end{aligned}$$

$H_{\mathbb{A}}^{2p}(X)(p)$ t **相对于 σ 的 Hodge 类**

$$\sigma^* t \in H^{2p}(\sigma X, \mathbb{Q})(p) \cap H^{p,p}(\sigma X).$$

$H_{\mathbb{A}}^{2p}(X)(p)$ t **绝对 Hodge 类**

$$\sigma: k \hookrightarrow \mathbb{C} \quad \sigma \quad \text{Hodge} \quad .$$

$$C_{\text{AH}}^p(\quad)$$

$$\mathbf{CV}_k : \mathcal{M}_k \rightarrow \mathcal{M}_k, \quad (h(X), X \in \mathbf{V}_k)$$

$$\mathrm{Hom}(h(X), h(Y)) = \mathrm{Mor}^0(X, Y).$$

$$h(X \sqcup Y) = h(X) \oplus h(Y), \quad h(X \times Y) = h(X) \otimes h(Y).$$

$$\mathbb{1} = h(\mathrm{pt}).$$

$$\mathbf{CV}_k \text{ (effective) motive } \dot{\mathbf{M}}_k^+.$$

()

$$\mathbb{P}^1, \quad h(\mathbb{P}^1) = h^0(\mathbb{P}^1) + h^2(\mathbb{P}^1) = h(\text{pt}) + L.$$

$$H(L) = \mathbb{Q}(-1),$$

$$\text{Hom}(M, N) \xrightarrow{\sim} \text{Hom}(M \otimes L, N \otimes L)$$

motive M, N .

L

:

motive \dot{M}_k :

① $(M, m), M \in \dot{M}_k^+, m \in \mathbb{Z};$

②

$$\text{Hom}_{\dot{M}_k}((M, m), (N, n)) = \text{Hom}_{\dot{M}_k^+}(M \otimes L^{r-m}, M \otimes L^{r-n})$$

$r \geq m, n;$

③

\dot{M}_k^+

.

\dot{M}_k
 $H^r(X)$ Hodge

 $\text{Mor}^0(X, X)$
 $\text{Hom}_{\dot{M}_k^+}((h(x), p), (h(x), p)) = p\text{Mor}^0(X, X)p$
 p

Jannsen 1992

Lemma 2



$()$
 \dot{M}_k
 (rigid)
 L
 $h(X), X \in \mathbb{V}_k, X \quad n,$

$$h(X)^\vee = h(X)(n),$$

 $\delta \quad [\Delta_X], \quad \text{ev}$
 $p \in \text{Hom}(h(X), h(X)),$
 ${}^t p \quad h(X)^\vee$

$$(h(X), p)^\vee = (h(X)^\vee, {}^t p).$$



$$\dim h(X) = \chi(X), \quad X \in \mathcal{V}_k,$$

$$\dot{\phi} : M \otimes N \rightarrow N \otimes M, \quad \dot{\phi} = \bigoplus \dot{\phi}^{r,s}, \quad \dot{\phi}^{r,s} : M^r \otimes N^s \rightarrow N^s \otimes M^r$$

 \dot{M}_k

$$\phi : M \otimes N \rightarrow N \otimes M, \quad \phi = \bigoplus \phi^{r,s}, \quad \phi^{r,s} = (-1)^{rs} \dot{\phi}^{r,s}.$$

 $M_k,$
正确的 motive 范畴.

$$H_\ell, H_{\text{dR}}, H_\sigma$$

xÊ X

—V Š 8 T +» -¿ Óf n- :V! V %²
cÁ XđY Ô « Ú¾”

$$() = /2i \quad V = \sum_{n=1}^{\infty} a_n \cdot 1^{n-1} + \dots + (-1)^n a_n :$$

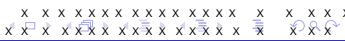
éÄ a_i = i` i ^i V X

šç X

› ,gç” # /2i V = i` n ^n V X
1¿• ñ -Ä

$$f_i = \frac{X}{(i \quad B / ^n i)} = S_n / (S_i \quad S_{n-i})$$

a_i = i` f_i ^n V :



x Ê X

—M ©%² ™ O 8 T + » -f :M ! M %² cÁ -
 éÄY "ÿèy ,
 X (1)ⁿ i`f n ^n M tⁿ X i`f m avKM t^m = 1:
 n 0 m 0

š ç X

1 ¥3† 'f J• -éQ-šç+`P3 M.. f I-®Ä
 ^nM avkⁿM = ^nM avkⁿM:
 $\frac{0 \ n \ 1}{n \ 2p2M}$ $\frac{0 \ n \ 1}{n \ Q//}$
 ðYY Ö¥ U£àš, > ¥0Ö` © V,
 0!^ !M !^ ! 1M avkM ! ! avkM ! 0;
 •w^aËß¥~çF•g "ã -Yšç ©%² 3`ç X



š ø ù v x

+ 9 3 ... f n - " p_n = $\frac{1}{n!} 2^{S_n}$; q_n = $\frac{1}{n!} 2^{S_n} (1)^{b;M}$ -
 ã ð + • a v K y ^ n 3 † ^ « X
 ð Y £

¿ X X Á 0 Ö ` ã ð q_n p_m q_{n+1} p_m y q_n p_{m+1} q_n p_m X
 £ ± ñ [Ö ` ® 0 X
 4 Ü - i Ü + f y = œ = [" , [%² @ f e C © 3 † ^ X
 Â e ... Y - ò , □

- ò X

$$q_n p_m = \frac{n(m+1)}{m+n} q_n p_{m+1} q_n p_m + \frac{m(n+1)}{m+n} q_n p_m q_{n+1} p_m :$$



Ô`Y xÊ±ñ

P - X

$$-V \textcircled{\hat{i}} \check{S} 8 + \gg - \textcircled{V} c\acute{A} - \acute{e}\acute{E} \`$$

$$x\hat{E} = \ddot{A} \text{ "èy ,$$

$$/2i1 \quad t \quad V \quad 1 = \frac{X}{n \ 0} \quad i \` \quad n \quad a \ v \ K \ V \quad t^n:$$

R ò U Y w2iáf Ê

$$Z(X;t) = \frac{X}{n \ 0} \quad i \` \ 6 _q \`^n \ a \ v \ K \ h(X) \ t^n;$$

$$= \frac{X}{n \ 0} \quad i \` \ 6 _q \` \ h(a \ v \ K \ X) \ t^n:$$



U Y # K Q i B p B á y w 2 i á f 2 0 f " % 2 8 3 t ' f ,

$$Z(X;t) = \sum_{n=0}^{\infty} h(a v K X) t^n$$

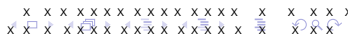
$$h p ñ i` h(X) = \sum_i^P (1)^i i` h^i(X) -$$

n ç y n ĩ È ç © , c X

4 - + ` p ê c Á K_0(J_k)! R - ® U Y < ' ç w 2 i á f X

ò " 2 i i B b = i` i j h () -

Ô « 0 ç > Q / ; h^{p;q} ^ X



† ò : ° i i b + ? ' 2 ,

+ 9 % 2 2 n Ó W f T M X - P_{x;y}(X) n P

+ f > Q / p 2 ÿ è á f - /

$$P_{x;y}(X) = \sum_{n \ i;j \ n} h^{n+i;n+j}(X) x^i y^j :$$

— S % 2 ¼ μ - S⁽ⁿ⁾ = a v K S = Sⁿ / S_n -
 S^[n] n P S n μ > B H # ' 2 ` i X

$$h^{i;j} = h^{i;j}(S) - \acute{e} \ddot{A} \hat{^} " ,$$

