

# NEW RESULTS ON THE REAL JACOBIAN CONJECTURE

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**The real Jacobian conjecture said:**  $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$   
 $x \in \mathbb{R}^2 \quad \det(DF(x)) \neq 0 \implies F^{-1}(x) \text{ is a polynomial map.}$

This conjecture had a negative answer by Pinchuk in 1994. Now several authors look for adding an additional assumption to the fact that  $\det(DF(x))$  is different from zero for all  $x \in \mathbb{R}^2$ , in order that the conjecture holds.

The next two theorema are proved using qualitative theory of the ordinary differential equations in the plane. More precisely in the talk we will show how the Poincaré compactification of polynomial vector fields, and the Poincaré–Hopf Theorem are used for proving the next two results.

**Theorem 1.**  $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$   
 $\det DF(x) \neq 0 \implies F^{-1}(x)$  is a polynomial map  
 if  $F(0,0) = (0,0)$  and  $F$  has no other critical points.

**Theorem 2.**  $F = (f, g) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$   
 $\det DF(x) \neq 0 \implies F^{-1}(x)$  is a polynomial map  
 if  $F(0,0) = (0,0)$  and  $F$  has no other critical points.